An Introduction to Robot SLAM (Simultaneous Localization And Mapping)

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Thanks Amy Briggs and Daniel Scharstein who are largely responsible for my Computer Science education.

Also, thanks Mom and Dad.

Contents:

1 Introduction

SLAM is one of the most widely researched subfields of robotics. An intuitive understanding of the SLAM process can be conveyed though a hypothetical example. Consider a simple mobile robot: a set of wheels connected to a motor and a camera, complete with actuators—physical devices for controlling the speed and direction of the unit. Now imagine the robot being used remotely by an operator to map inaccessible places. The actuators allow the robot to move around, and the camera provides enough visual information for the operator to understand where surrounding objects are and how the robot is oriented in reference to them. What the human operator is doing is an example of SLAM (Simultaneous Localization and Mapping). Determining the location of objects in the environment is an instance of mapping, and establishing the robot position with respect to these objects is an example of localization. The SLAM subfield of robotics attempts to provide a way for robots to do SLAM autonomously. A solution to the SLAM problem would allow a robot to make maps without any human assistance whatsoever.

The goals of this project are to explore the motivation behind current SLAM techniques and to present multiple effective solutions to the SLAM problem in a way that is accessible to individuals without a background in robotics. The solutions considered in this thesis will include the Kalman Filter (KF), which is the backbone of most SLAM research today. Next we will consider FastSLAM which utilizes the KF and incorporates particles. Finally, will discus DP-SLAM which utilizes the particles from the FastSLAM algorithm and develops a new way of storing information about the environment.

These particular algorithms were chosen because they build off one another. The popular and powerful KF will be used as a stepping stone on the way to the more state-of-the-art FastSLAM and DP-SLAM algorithms. This pattern of building up to more complicated algorithms is reflected in the structure of the thesis as a whole as well as the individual sections. In the development of the SLAM problem we will first examine localization and mapping individually to better understand how and why we should find a simultaneous solution. The section which develops the Kalman Filter will begin by solving a very simple one-dimensional example and proceed to complicate the situation until we arrive at a full-fledged SLAM-based Kalman Filter. The discussion of the Kalman Filter, in turn, will both motivate and elucidate the steps involved in both DPSLAM and FastSLAM.

It is the hope of this author that this will allow anyone who is interested to arrive at a deep understanding of these advanced SLAM techniques without ever losing their footing.

1.1 Applications

If a solution to the SLAM problem could be found, it would open the door to innumerable mapping possibilities where human assistance is cumbersome or impossible. Maps could be made in areas which are dangerous or inaccessible to humans such as deep-sea environments or unstable structures. A solution to SLAM would obviate outside localization methods like GPS or man-made beacons. It would make robot navigation possible in places like damaged (or undamaged) space stations and other planets. Even in locations where GPS or beacons are available, a solution to the SLAM problem would be invaluable. GPS is currently only accurate to within about one half of a

meter, which is often more than enough to be the difference between successful mapping and getting stuck. Placing man-made beacons is expensive in terms of time and money; in situations where beacons are an option, simply mapping by hand is almost always more practical.

1.2 A History of Robot SLAM

SLAM is a major, yet relatively new subfield of robotics. It wasn't until the mid 1980s that Smith and Durrant-Whyte developed a concrete representation of uncertainty in feature location (Durrant-Whyte, [2002]). This was a major step in establishing the importance in finding a practical rather than a theoretical solution to robot navigation. Smith and Durrant-Whyte's paper provided a foundation for finding ways to deal with the errors. Soon thereafter a paper by Cheesman, Chatila, and Crowley proved the existence of a correlation between feature location errors due to errors in motion which affect all feature locations (Durrant-Whyte, [2002]). This was the last step in solidifying the SLAM problem as we know it today. There is a lot more we could say about the history of breakthroughs that have been made in SLAM but the ones we have mentioned are perhaps the most important (Durrant-Whyte, [2002]), so instead it is time to take a closer look at the SLAM problem.

1.3 Components of a Process State

Several important terms regarding components of the world state will be referred to repeatedly throughout this project. A short time is devoted here to solidifying precisely what some of these ideas are.

The term "feature" is used in this project to refer to a specific point in the environment which can be detected by a mobile robot. The term "map" will always refer

to a vector of feature location estimates. In practice, features range from distinctive patterns artificially placed/scattered in the robot's environment to naturally occurring cracks or corners chosen by the mobile robot. (For the purposes of this project, a feature and a beacon serve exactly the same purpose and will be considered essentially the same thing.) Selecting, identifying, and distinguishing features from one another are nontrivial problems (Trucco, Verri, [1998]), but they reside outside the scope of this project. We will assume that features are distinguishable and identifiable when the robot scans the appropriate location. Once a feature is found, however, it is not permanently visible to the robot; whenever the robot changes its position it loses track of all features it has located and will need to find them again.

We will also assume that when a robot scans a feature it is capable of estimating the distance between the feature and itself as well as what direction the feature is in. Distance can be estimated in a number of ways. If a feature is of known physical dimensions, the perceived size of the feature can indicate distance. Another method of estimating depth is to use multiple cameras and employ stereo vision techniques to triangulate distance (Scharstein, [2006]). Distance estimation, like feature tracking, is a nontrivial problem which resides outside the scope of this project.

The terms "pose" and "robot pose" are used synonymously to refer to how the robot is positioned. This includes any variables pertaining to the robot itself. In a simple case the pose is a vector containing the x and y coordinates of the robot along with the orientation, theta. In reality, the pose of the vehicle can be more complicated. Variables pertinent to a mobile robot on non-level terrain, for example, will include pitch and roll

as well as theta. Lastly, a "state" consists of the most recent pose and map estimated by the robot. We are now ready to develop the SLAM problem.

1.4 The SLAM Problem

In section 1.2 *History of Robot SLAM* it was noted that one of the major developments in SLAM research was proof that errors in feature location acquired by agents are correlated with one another. The correlation exists because an error in localization will have a universal effect on the perceived location of all features (we will discuss this in more detail in the chapter on the Kalman Filter). Understanding and utilizing the relationship among errors in feature locations and robot pose is at the core of SLAM research; it is the major motivation behind solving localization and mapping concurrently. This chapter will now consider localization and mapping each individually to develop a better understanding the SLAM problem and further illuminate the motivation behind finding a simultaneous solution.

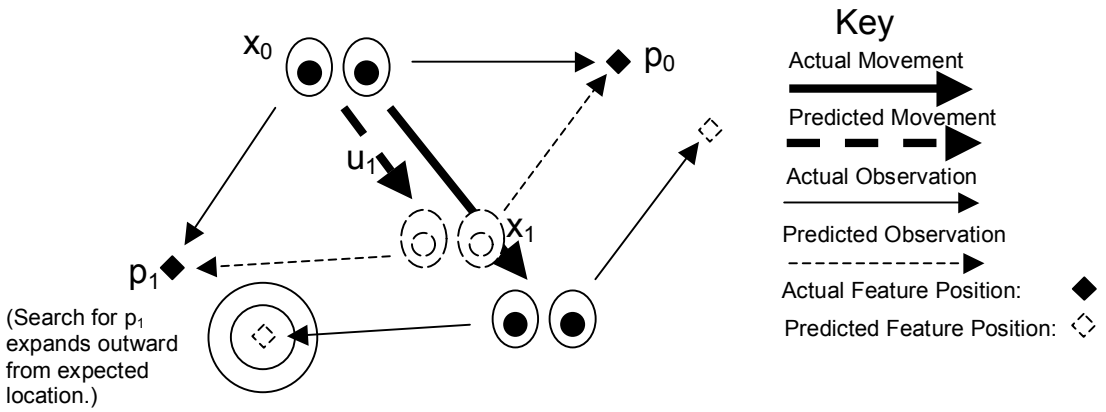
Works by Maybeck, Durant-Whyte, Nebot, and Csorba all helped inspire this portion of my thesis, but the development of SLAM problem as it appears here is (as far as I know) unique. It represents a precursor to the works of the authors mentioned above, and is written to provide motivation for various steps taken in their respective works. Few specific citations are made, but this section would not have been possible without understanding gained from these authors.

In particular, many of the specifics of the localization problem and the mapping problem presented here are elaborate on diagrams created by Durant-Whyte.

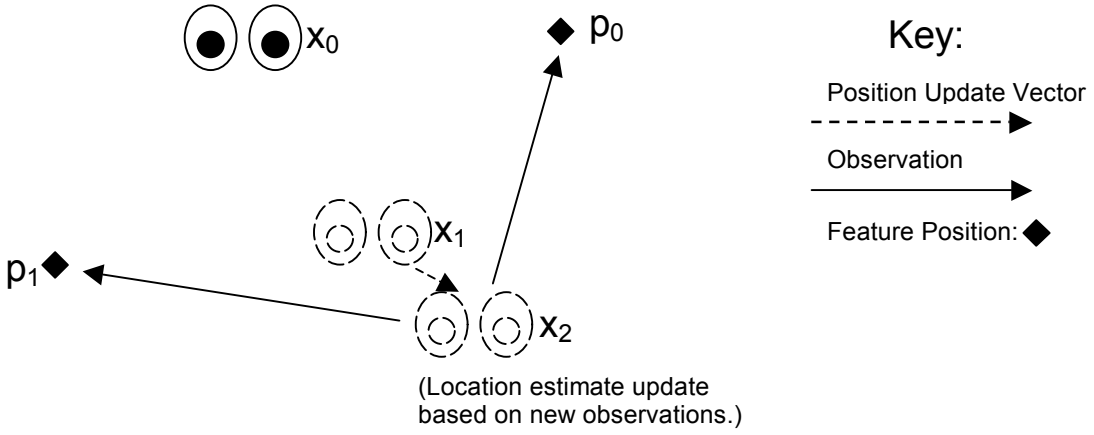
1.4.1 The Localization Model

Suppose we have a mobile robot in an environment which contains beacons—points in the environment with a known exact location and which (once acquired) the robot can calculate its distance and direction from. Further suppose that after initialization the robot cannot assume to know its exact location or orientation; due to inaccuracies of the actuators it must attempt to extrapolate this information from the beacons. Our robot could theoretically navigate such an environment without too much difficulty even if the robot's actuators were not completely accurate.

Imagine that the robot starts in some known location, x_0, and has knowledge of a set P containing several feature locations. (Notation is specified in detail in section 2.6.1 *Adapted Notation for Multiple Dimensions*.) The world is exactly as our robot perceives it because the robot's (noisy) sensors have not been needed yet. When the robot attempts to move a given distance in a certain direction (i.e. the movement vector u_1) to location x_1, it will actually move along some other vector to some location which is nearby x_1. The beacons need to be relocated to determine the new robot pose. If the actuators were completely accurate the beacons could be reacquired by assuming they have moved inversely to u_1. Because the actuators are imprecise, however, the robot must search for the beacons. The search is facilitated by beginning near the expected location of the beacon and expanding outward. The following diagram presents the situation so far:



Notice that x_1 as well as the movement vectors correspond to estimates rather than ground truth. (This convention will be dropped briefly in section 1.4.2 *The Mapping Model* but will be readopted once we begin to discus Simultaneous Localization and Mapping; a SLAM algorithm will never have access to its actual position or movement vector and will therefore have no reason to name it.) Once the beacons are reacquired, a new robot location estimate, x_2, can be made, as shown below:



Over time, the estimate of the robot pose will never diverge because the beacon locations are fixed and known absolutely; the robot pose is always calculated from the same correct data. We use beacons rather than features in this example simply because

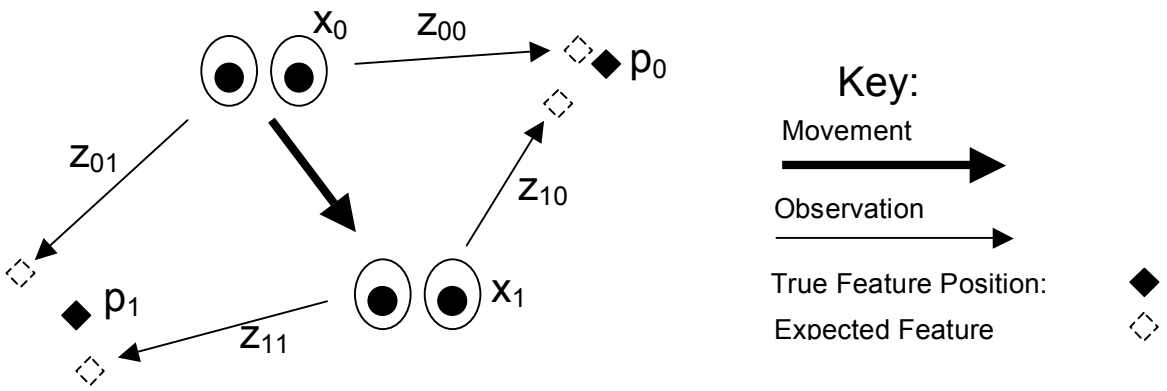
we would be more likely to know the absolute location of beacons than naturally-occurring features. Note that the situation would be essentially the same if the agent began with an accurate feature map rather than beacons. The only difference would be that the agent would need to reacquire features from camera images rather than simply finding beacons.

1.4.2 The Mapping Model

Mapping, in a sense, is the opposite of the situation described above. In mapping, we assume the agent knows exactly where it is at all times—that is, we assume we either have perfect motion sensors, or an error free GPS. What the agent lacks in the mapping context is a way of knowing the absolute location of features. Notice that in contrast to localization, the agent does not begin with an accurate world view; the agent can locate features to build a map, $z_0 = \{z_{00}, z_{01} \ldots z_{0n}\}$ where z_{ij} is read, "the j^{th} feature estimate at time step i". Of course, z_0 will only contain approximations of the actual feature locations. When the agent moves, however, the map can generally be made more accurate.

Consider the robot from the previous section beginning at x_0 and moving along u_1 to x_1. The robot will once again need to acquire the set of features, just as it did in *The Localization Model*, but this time the perceived locations of the features will have shifted from their expected locations due to sensor inaccuracies rather than odometry inaccuracies. The situation is the same as before, however; the features can be relocated by searching near where the robot expects to find them. Once this is accomplished the robot will have built a new map, z_1, consisting of new locations of the features in z_0.

(Preservation of features is assumed here for simplicity; in reality some features are lost and new features are acquired.) The figure below demonstrates the process:



To generate a new map, z_2, which combines the information in z_0 and z_1 but contains only one instance of each feature, the robot can choose one of many options. One idea is to minimize the total distance between the two perceived locations of each feature j (these perceived locations are denoted z_{0j} member of z_0, and z_{1j} member of z_1) and the new expected location of feature i (z_{2j} member of z_2). This, in our two dimensional example, amounts to choosing any point on the line connecting z0i and z1i for each feature i. (Remember: z0i and z1i are the two conflicting perceived locations of feature i.) As new readings are taken the estimate will become more precise. This basic method may be sufficient in a purely mapping context, though it is not optimal. Another option is to minimize the squared distance rather than the total distance. In Chapter 2 T*he Kalman Filter*, we will see that this solution is quite promising.

Whichever method we choose for incorporating new sensor readings, it seems safe to assume that z_i will generally improve as i increases. As our robot moves and acquires more readings, the estimates in z_i will tend to become more and more accurate just as long as we make some assumptions about perception errors. In particular we must

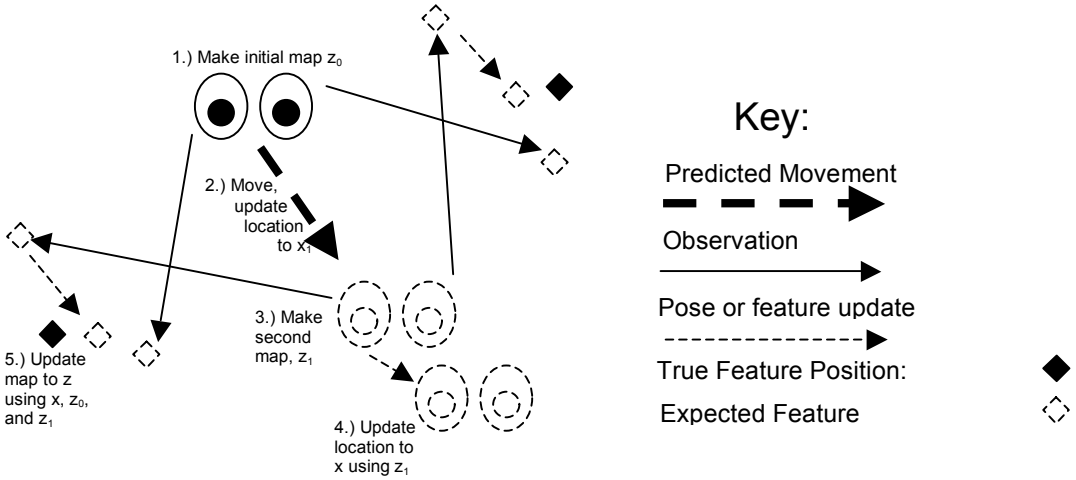
assume well calibrated sensors. If the sensor readings are routinely off to one particular side of a feature due to poor calibration, that feature can't be honed in on.

The assumption of proper calibration, along with other ideas from this section, will prove invaluable throughout this project.

1.4.3 The Simultaneous Problem

Each of the algorithms described above require something as input that is generally unavailable in practice. A localization algorithm outputs the robot pose but requires accurate feature locations (i.e. a map) as input every time the algorithm iterates. Conversely, a mapping algorithm generates a map but needs an undated pose as input. The fact that the (usually unavailable) input of each process is the output of the other suggests that the two problems are related, and could perhaps be combined into a single solution; it seems that if both processes are utilized together there is some hope for robot navigation without either an a-priori map or accurate GPS/odometry. One naïve solution is considered below.

Suppose a robot starts from some known origin. It uses the process described in 1.4.2 *The Mapping Model* to construct a preliminary map. It then moves to a new location and updates its expected location as described in 1.4.1 *The Localization Model*. Finally the newly calculated pose is used to once again generate a new map which is somehow combined with the original map as described in section 1.4.2. By repeating this process we can provide a map for input into the localization function, and a location for input into the mapping function. This is one possible solution to the SLAM problem, and though it is a naive one it is a good place to start. The following diagram demonstrates the basic SLAM idea; pay attention to the order in which the process is carried out:



One key thing to notice with this combined localization and mapping algorithm is that we do not provide completely accurate input to either the mapping or the localization components. In section 1.4.1 *The Localization Model* it was noted that the estimated vehicle location would not diverge from the actual vehicle location. This was possible because the robot pose was always estimated using accurate feature locations. Similarly, we assumed the mapping algorithm would converge only because the robot pose is always known exactly. Neither of these assumptions hold true in the SLAM context. Care will be taken later to find a way to guarantee convergence of the SLAM problem in a theoretical sense.

Another key thing to note in this preliminary solution is the significance in the order of the two steps, localization and mapping. In the version described above, the robot moves, updates vehicle pose, and then maps. This allows the robot to utilize the new vehicle pose to make a better map. On the other hand, if it instead moved, mapped, and then localized it could use the new map to make a better localization estimate. Either way, valuable new information gets ignored in the second step. Clearly the vehicle pose estimate and approximate map influence one another. This insight suggests a truly

simultaneous solution which takes into account the relationship between vehicle and feature locations. This leads us to a discussion of the first rigorous mathematical solution to the SLAM problem: the Kalman Filter.

2 The Kalman Filter

The Kalman Filter (or KF) was developed by R.E. Kalman, whose prominent paper on the subject was published in 1960 (Welch and Bishop, [2006]). The KF potential in specific localization and mapping problems was well known shortly after Kalman published his paper in 1960. However, the KF was not generally used to represent an entire world state (and was therefore not used as a complete solution to the SLAM problem) until 1988 (Castellanos, Neira, Tardos, [2004]).

The Kalman Filter is an algorithm which processes data and estimates variable values (Maybeck, [1979]). In a SLAM context, the variable values to be estimated will consist of the robot pose, and feature locations—i.e. the world state. The data to be processed may include vehicle location, actuator input, sensor readings, and motion sensors of the mobile robot. In other words, the Kalman Filter can utilize all available data to simultaneously estimate robot pose and generate a feature map. Under certain conditions, the estimates made by the Kalman Filter are very good; in fact, they are in a sense "optimal". Welch and Bishop explain that the state estimates provided by the Kalman Filter will use any available information to minimize the mean of the squared error of the estimates with regard to the available information (Welch and Bishop, [2006]). Another way of putting this is that the error in the state estimates made by the Kalman Filter are minimized statistically (Maybeck, [1979]).

2.1 The "Kalman" Idea

In section 1.4.2 *The Mapping Model* we considered choosing expected feature locations based on minimizing the squared error. This basic premise is the idea behind the Kalman Filter. In fact, if we start by creating a mathematical method of minimizing

squared error in a simple situation, we can then extend the method to be appropriate for a SLAM context in such a way that we arrive at an instance of the Kalman Filter. Keeping in mind that one of the goals of this thesis is to present solutions to the SLAM problem in an intuitive way, we now develop the mathematics behind the Kalman Filter in the method just described.

Section 2.4 *Developing a Simple Filter* will construct a simple Kalman Filter for a basic static estimation problem that is similar to but simpler than the SLAM problem. In the proceeding sections, the filter from section 2.4 will be adapted to describe a legitimate SLAM situation.

2.2 The Kalman Filter and SLAM

There are many reasons for why KF techniques are such a popular choice for SLAM researchers. As was mentioned previously, the KF is capable of incorporating all available measurements (Maybeck, [1979]) and providing a least squares estimate of a process state. When modeled carefully, this is precisely what SLAM algorithms attempt to do. Furthermore, as we saw in *The Slam Problem*, SLAM would be trivial if robot sensors were noise free; handling noise in the best way possible is the essential purpose of the KF, (Maybeck, [1979]). Finally, the KF works well in practice and so researchers continue to use it (Durant-Whyte, [2002]).

2.3 Concepts: White Noise, Gaussian Curves, and Linear Processes

A "least-squares" estimate is only useful when errors behave in a certain way. Consequently, the claim that the KF provides a least-squares estimate of a state is meaningful only if certain assumptions are made about the behavior of sensor errors. Two

assumptions made by the KF are that the noise has zero mean process noise and is Gaussian-distributed (Csorba, [1997]).

Process noise with zero mean, also called white noise, is noise which is uncorrelated between time steps—the KF operates under the assumption that any particular actuator or sensor error will not be indicative of the value of any future errors (Maybeck, [1979]). To better appreciate what this requirement implies, imagine a man throwing darts at a dartboard. In general, we would expect that, given long enough, the average x and y values of the man's throws we would be right on the bull's eye. The man might not even be great at darts, but he would not have a predilection for hitting a particular section of the dart board so his average shot will be quite good. In other words, his throws would be noisy, but the noise would be white.

For an example of non-white noise consider the following: suppose the dart thrower is playing with darts of a different weight than he is used to, and consequently tends to either under-throw or over-throw the darts (but we don't know which). In this case, if we knew y-value of one of the man's shots it would change our prediction for the next shot. For example, if the man under-throws a dart, we might suppose that the darts are heavier than he is used to and that he will likely under-throw the next dart at well. The KF white noise requirement is not one that can be completely guaranteed in practice for robot sensors, but it should be possible to make any non-white noise small enough to be negligible. In other words, non-white noise indicates fixable hardware or software problem, or some other potential to improve the process model (Csorba, [1997]). Thus, whiteness of noise in the SLAM process is achievable in a practical sense.

A Gaussian-distribution of noise is also achievable, if we are careful. In short, a Gaussian-distribution of noise means that at any given time the probability density of the amplitude of the system noise takes on a bell shaped curve (Maybeck, [1979]). According to the central limit theorem, a Gaussian curve arises whenever many independent and identically-distributed random variables are added together (Csorba, [1997]).

To make this idea clearer, let's go back to the dart board example. Suppose the dart thrower throws for a long time and afterwards we want to graph the number of darts hits versus the x value of the dart board. If the thrower's aim is affected by lots of little factors (a tiny breeze, barely over-extended arm, slightly cocked wrist, and other small factors) we can expect the graph to take on a particular shape with most of the hits centered around the bull's-eye, and fewer and few hits as we move away from the bull's-eye. This particular shape is a Gaussian curve. Furthermore, the variance of the darts (the average squared distance between a dart and the bull's-eye) uniquely defines this Gaussian curve (Maybeck, [1979]). Consequently if we know the variance of the dart thrower (based on many, many of his throws), and we know the x value of just one dart x_d, the Gaussian curve defined by the variance centered at x_d would define the statistical likelihood of the location of the bull's-eye.

We can do something similar with our robot's sensor. For example, if we do extensive testing of a distance sensor to determine the variance of the readings made by the sensor, a single sensor reading will define a Gaussian shaped curve corresponding to the likelihood of the true distance. The KF relies on being able to do this, and

consequently we must be able to assume a Gaussian distribution of any noise we do not explicitly account for.

Csorba argues that a Gaussian distribution of noise is possible for robot sensors (Csorba, [1997]). Large error sources in the SLAM model can be modeled for explicitly, and the remaining errors are small and arise from many independent sources (Csorba, [1997]). Csorba does not assert that these small perturbations are identically-distributed, but implies that the large number of small errors and independent sources of these errors justifies the application the central limit theorem in practice (Csorba, [1997]). Thus the probability density will be close enough to a Gaussian curve for practical purposes.

One final assumption made by the Kalman Filter actually can pose a problem for SLAM researchers. This assumption is that the process model is linear, which is untrue for most complex SLAM situations. The linearity of a process is a measure of its complexity. In a swept lab with a hard floor, the process model for a mobile robot might be approximately linear. If the robot is in an outdoor environment where debris might block a wheel or if the robot's power reserves drop so the actuators lose potency then the process model would be decidedly nonlinear. Suppose we can somehow represent the control inputs as a vector of values. We will call a process model linear if it is possible to describe the most likely position of our robot as a linear function of those values added to the previous position.

For example, suppose the actuator input is a single value that corresponds to power sent to the motor for a constant amount of time. If we can always multiply the value of the actuator input by some constant and add it to the previous position to get the most likely new position of the robot, then the process model is linear. Otherwise, as is

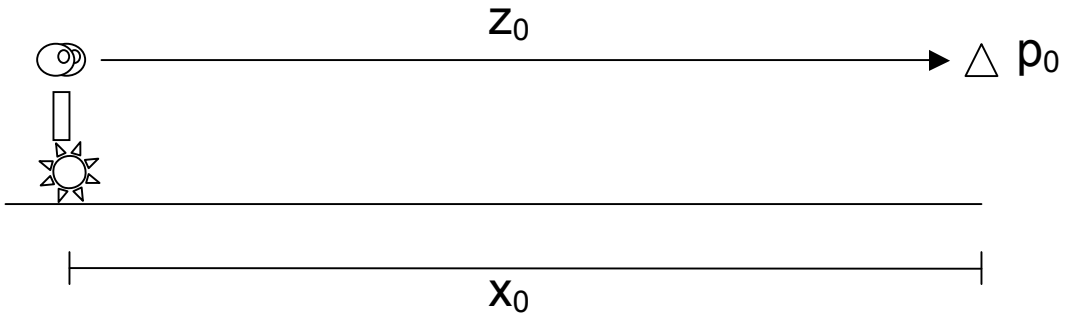
very frequently the case, the process model is nonlinear. The Kalman Filter is still useful in many situations where the process model is nearly linear, but later we will want a KF solution to nonlinear process models. The solution will be to develop the Extended Kalman Filter.

The Extended Kalman Filter (EKF) is used extensively in localization and mapping and represents the backbone of most SLAM algorithms (Durant-Whyte, [2002]). The EKF, the primary version of the Kalman Filter used in SLAM research, is very similar to the un-extended Kalman Filter. The claims made thus far concerning the KF hold true for the EKF, and by the time we finish describing the KF, the extension to the EXF will be trivial. The basic difference between the KF and the EKF is that the KF handles only linear process models, whereas the EKF is designed to handle more complex non-linear process models and is therefore more useful in SLAM research (Durant-Whyte, [2002]). For now, we will be concerned with simple linear process models and will not worry about the EKF until the end of the chapter.

2.4 Developing a Simple Filter

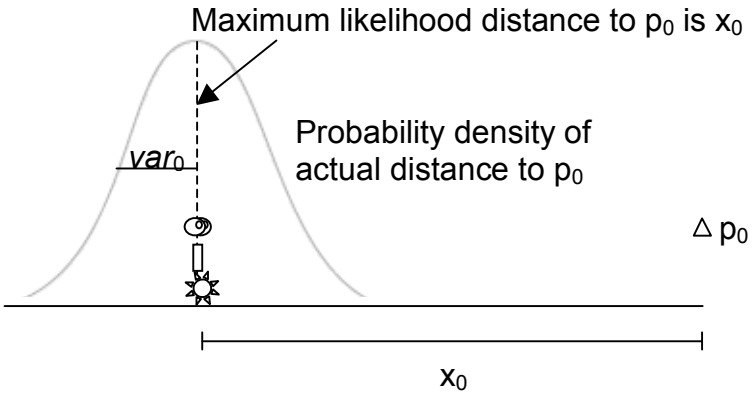
The following mathematical steps are based on conclusions developed in Peter S. Maybeck's Stochastic models, estimations, and control chapter 1. All numbered equations originally appear in his work in some form. The circumstances described here, however, have been changed to be more pertinent to understanding the KF from a robot SLAM point of view. Additionally, all diagrams as well as all non-numbered steps and derivations are original to this thesis have been incorporated to make the development of the Kalman Filter easier to follow.

To keep things as simple as possible, let us first consider a 1-dimensional example. Suppose a mobile robot is restricted to a line on the ground. It makes an observation z_0 and identifies a single feature p_0, and uses this observation to make an estimate of the distance between the robot and the feature x_0. (Notice how these definitions have changed subtly from Chapter 1 *Introduction*; these new definitions properly reflect the simplified situation.)



Notice that the image of the robot in this diagram has changed from previous sections. This is to convey visually that we are looking at the robot from a side view and to remind the reader that we are now dealing with a 1-dimensional problem. This will be a useful way to distinguish multidimensional diagrams from 1-dimensional ones.

Suppose prior testing has given us an estimate for how accurate our sensors are. It would be reasonable to assume that the probability density will be approximately Gaussian as explained in section 2.3 *Concepts: White Noise, Gaussian Curves, and Linear Processes*. We may now claim that, all things being equal, our estimate of x_0 is more likely than any other estimate, and has a particular expected squared error or variance, var_0 (for more information on what this means, see section 2.3 *Concepts: White Noise, Gaussian Curves, and Linear Processes*).



Instead of immediately moving, let us suppose our robot makes a second reading with some other sensor; this will allow us to develop a way of incorporating new information before the situation gets complicated. The new reading, z_1, yields a new vehicle estimate of x_1 with a new variance var_1. Now, where shall we suppose our robot is: x_0 or x_1? We could just trust the sensor with the better accuracy—i.e. lower variance—but this would ignore valuable information provided by the other sensor. We could average the two estimates, but when var_0 != var_1 we know one sensor is more accurate than the other and therefore deserves more weight.

We will refer to the next position estimate of our robot as x. In pursuit of finding an appropriate way of weighting each estimate, let's consider updating x in the following way:

$$x <- (var_1/(var_0 + var_1))\, x_0 + (var_0/(var_0 + var_1))x_1 \qquad \text{[Equation 1]}$$

This should make good intuitive sense. Suppose var_1, the variance for the x_1 estimate, is higher than var_0. More weight will be given to x_0, which is desirable because we know x_0 to be the estimate based on the more accurate sensor. This method allows us to weight estimates proportionally to their variance (or equivalently, expected squared

error). It should therefore come as no surprise that the resulting value is the minimization of the squared error with regard to the previous estimates.

Now that we have a new estimate, x, let's consider the variance, *var*, of x. We should expect that *var* will depend on the previous two variances, but it should be less than either of them, i.e. $var < \min(var_0, var_1)$. If this is not the case, if $var > var_0$ for example, then by incorporating information from the second sensor we would have actually lost information. As long as the sensors are properly calibrated this should not happen. Fortunately, as will be explained in 2.3 *Concepts: White Noise, Gaussian Curves, and Linear Processes*, proper calibration of sensors is reasonable to presume.

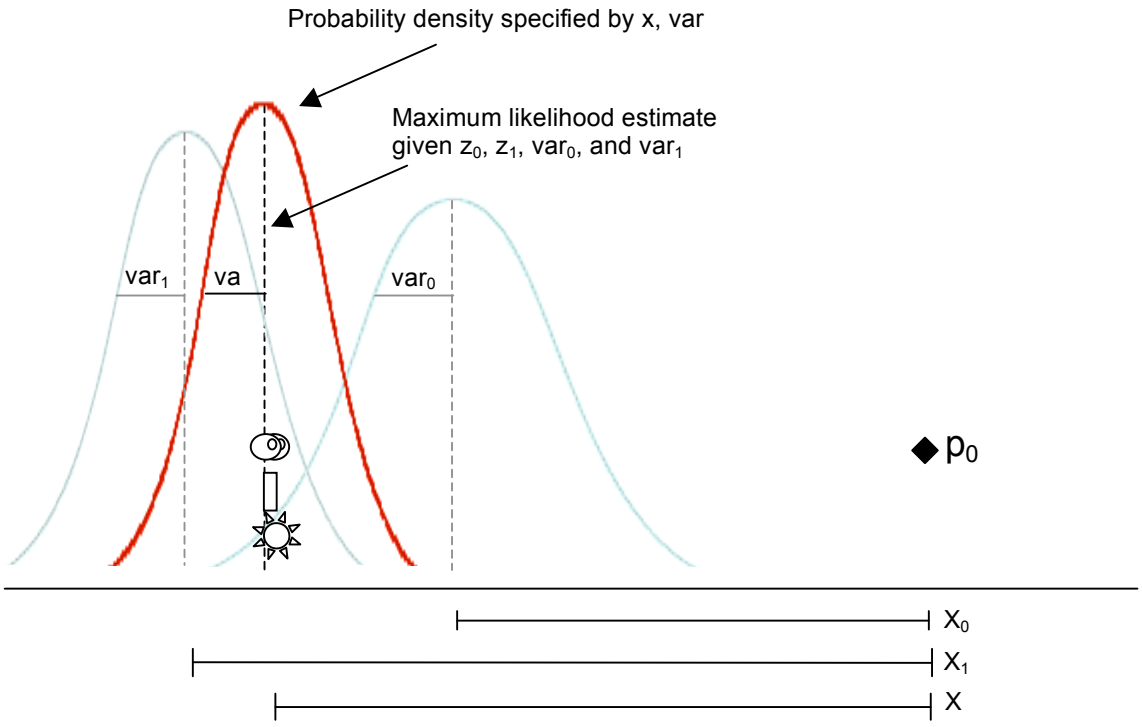
One would also expect *var* to experience the quickest (proportional) improvement over the previous variance if the two robot sensors are equally accurate. That is, $(\min(var_1, var_0) - var) / (var_1 + var_0)$ is maximized when $var_1 = var_0$. To see this, imagine two independent sensors measuring the same distance. If the two sensors are equally accurate we should be able to get a significantly better estimate (on average) by combining the readings then by picking one in particular. If one sensor is much more accurate than the other, on the other hand, not much information is likely to be gained by incorporating the inaccurate sensor (although there should be some small information gain if the readings are weighted appropriately).

The new variance can be statistically determined as follows:

$1/var \leftarrow 1/var_1 + 1/var_0$ [Equation 2]

Simple examination shows that this update equation exactly models the way we expected the variance to behave: $var < \min(var_1, var_0)$ for any variances *var1* and *var0*, and $(\min(var_1, var_0) - var) / (var_1 + var_0)$ is maximized whenever $var_1 = var_0$.

The following diagram demonstrates how the bell shaped probability density specified by x_0 and var_0 combines with the probability density specified by x_1 and var_1 as specified by [Equation 1] and [Equation 2]:



The update equations we have described in this section are now already very close to a very basic application of the Kalman Filter. All that remains to be done is some algebra and a redefinition of terms. First, to set up the equations as appropriate for a recursive SLAM context, let's generalize the update processes to be valid for any time step, i:

[Equation 1] becomes:

$$x_i = (var_{i-1}/(var_s + var_{i-1}))\, x_s + (var_s/(var_s + var_{i-1}))x_{i-1} \qquad \text{[Equation 3]}$$

Where:

x_i is the updated location estimate at time i,

x_s is a location estimate based on a new sensor reading,

x_{i-1} is last location estimate (i.e. the best guess location based on the i-1 sensor readings we have taken so far),

var_{i-1} is the variance of x_{i-1}

var_s is the variance of x_s

With the equation in this form we can see how to incorporate as many independent sensor readings as we want as long as we can continue to update the variance of the last estimate. (Updating variance will be addressed shortly.) Now let's perform some rearrangements to make our update equations reflect standard Kalman Filter format. We begin with [Equation 3]:

$$x_i = \frac{var_s * x_{i-1}}{(var_s + var_{i-1})} + \frac{var_{i-1} * x_s}{(var_s + var_{i-1})}$$

$$x_i = \frac{(var_{i-1} + var_s)\, x_{i-1}}{(var_s + var_{i-1})} - \frac{var_{i-1} * x_{i-1}}{(var_s + var_{i-1})} + \frac{var_{i-1} * x_s}{(var_s + var_{i-1})}$$

$$x_i = x_{i-1} - \frac{var_{i-1} * x_{i-1}}{(var_s + var_{i-1})} + \frac{var_{i-1} * x_s}{(var_s + var_{i-1})}$$

$$x_i = x_{i-1} + \frac{var_{i-1}}{(var_s + var_{i-1})} * [x_s - x_{i-1}]$$

$$x_i = x_{i-1} + K_i * [x_s - x_{i-1}] \qquad \text{[Equation 4]}$$

where

$$K_i = \frac{var_{i-1}}{(var_s + var_{i-1})} \qquad \text{[Equation 5]}$$

Presenting the update process in this way elucidates what's happening to our location estimate. The estimate x_i is just x_{i-1} shifted according to the difference between

x_{i-1} and some new sensor derived estimate x_s. The new estimate x_s is weighted according to K_i which relates the variance of the new estimate to the variance of the old estimate. [Equation 4] is in proper Kalman Filter form (Maybeck, [1979]). All that remains now is to properly present the variance update equation. Generalizing [Equation 2], we have:

$$1/var_i = 1/var_s + 1/var_{i-1}$$

Now we derive the Kalman Filter version of the variance update equation:

$$var_i = \frac{1}{(1/var_s + 1/var_{i-1})}$$

$$var_i = \frac{1}{(var_s + var_{i-1}) / (var_s * var_{i-1})}$$

$$var_i = \frac{var_s * var_{i-1}}{var_s + var_{i-1}}$$

$$var_i = \frac{var_{i-1}^2 + var_s * var_{i-1}}{var_s + var_{i-1}} - \frac{var_{i-1}^2}{var_s + var_{i-1}}$$

$$var_i = var_{i-1} * \frac{(var_{i-1} + var_s)}{(var_s + var_{i-1})} - \frac{var_{i-1}^2}{var_s + var_{i-1}}$$

$$var_i = var_{i-1} - \frac{var_{i-1}^2}{var_s + var_{i-1}}$$

Finally, using [Equation 5], we arrive at the following equation:

$$var_i = var_{i-1} - K_i * var_{i-1} \qquad \text{[Equation 6]}$$

Equations 4, 5, and 6 together are the Kalman Filter solution to the presented static estimation problem (Maybeck, [1979]). We should take a moment here to appreciate the power of the tool we have just developed. We can alternate between making an optimal least-squares estimate of a distance using [Equation 4] and using [Equation 6] to represent our exact confidence in that estimate optimal. Furthermore, at time i, our estimate will be the optimal guess based on i sensor readings and the estimate

can be determined in *constant* time (in this static estimation problem). The fact that the complexity of the KF does not increase with the time index combined with the fact that the estimates made by the KF are optimal make the KF an extremely useful tool for SLAM researchers.

We still have a fair amount of work remaining in terms of developing a KF we can use for SLAM as we have defined. The static estimation problem above is simpler than SLAM situations in three significant respects:

* First, the robot in the above example never actually moves. We will need to account for vehicle translation and error in translation. (This will be relatively simple and is handled in the next section.)

* Second, the Kalman Filter we have developed so far only considers vehicle location with respect to a single feature. With multiple features we need to update multiple values after every reading instead of just one.

* Third, the environment in the above example is one-dimensional. Any practical SLAM environment will have at least two dimensions which need to be accounted for.

The solution we have developed is a good first start, however. We worked our way up to a legitimate instance of a Kalman Filter without making any unintuitive steps. Vectors and matrices will be needed to properly account for the last two considerations, but the basic operations and steps will be essentially the same as those in the Kalman Filter we developed in this section.

2.5 Developing a Kalman Filter for a Moving Robot

We ended the previous section by noticing three concerns, one of which is that we need account for robot motion. We will now adapt our Kalman Filter to handle motion.

In section 2.4 *Developing a Simple Filter*, we were able to make progressively better least-squares estimations by keeping track of and updating just two values: a location estimate x_i and our confidence in that estimate var_i. This remains true here; to account for control inputs made by the robot, all we need to do is figure out how control inputs affect these two values.

Durant-Whyte suggests considering vehicle control input in a two dimensional environment to be a vector of two values: a velocity and a heading (Durant-Whyte, [2002]). Strictly speaking, this is already a simplification because a mobile robot will have direct control over acceleration, not velocity. The same simplification will be made here as modeling the physics behind robot SLAM is not a goal of this thesis. To simplify the situation further, let's suppose a control input simply consists of a particular velocity, u_i. (It would be redundant to specify a direction in a one-dimensional environment.) In particular, u_i refers to the control input which brought the robot from the last expected location x_{i-1} to its current expected location x_i for any given time step i.

Notice that if our best location estimate is x_{i-1} and we move at an expected velocity u_i for a length of time T, our "best guess" or expectation of the new location estimate will just be x_{i-1} plus our expected movement vector. Our motion model, an adaptation of motion models from Durant-Whyte, Maybeck, and Csorba, is therefore:

$$x_i = x_{i-1} + u_i * T \qquad \text{[Equation 7]}$$

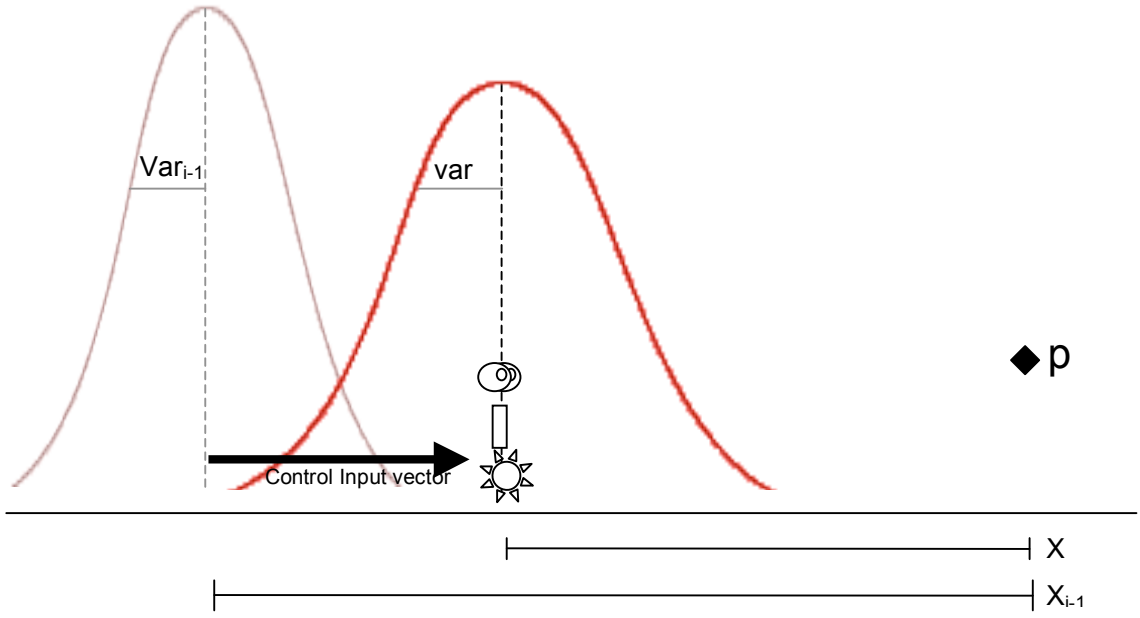
Where T = TimeValue(i) - TimeValue(i - 1), or in other words, T is equal to the time that has passed since the robot began moving from location x_{i-1}.

This is all we need to do to update the location estimate x_i; all that remains is to determine how moving ought to change the confidence we have in our estimate, var_i.

Notice that var_i should depend on var_{i-1} (the confidence we enjoyed before control input u_i) as well as var_u (the variance in the error of our actuators) weighted proportionally to T. Also notice that var_i should be greater than either var_{i-1} or var_u as moving decreases our expected accuracy. These two pieces of information nearly specify the update equation. Statistics tells us that the true variance update equation is:

$$var_i = var_{i-1} + var_u * T \qquad \text{[Equation 8]}$$

We have now very clearly specified how moving affects the state. The conceptual effect of Equations 7 and 8 on the variance and expected location is illustrated below; notice that the probability density widens, which indicates that after a move we are less sure of where we are:



Now that we have specified update equations given control inputs, our robot has all the tools it needs to make a move; it can appropriately update its expected location and variance using Equations 7 and 8 and proceed to continue enhancing its location estimate

as normal by taking new feature readings. [Equation 8] was the last step in implementing a Kalman Filter for our 1-dimensional environment. It should now be clear that non-divergence of the SLAM problem is possible. In this particular 1-dimensional environment, assuming we take a sensor reading after every move, we can see that as long as $K_i * var_{i-1}$ is on average greater than or equal to $var_u * T$, over time our estimate will always get better or stay the same.

For reference, the four update equations are repeated here with their original equations numbers:

Kalman Filter equations for 1 dimension and 1 feature:

State Update Given Sensor Input:

$$x_i = x_{i-1} + K_i * [x_s - x_{i-1}] \quad \text{[Equation 4]}$$

Uncertainty Update Given Sensor Input:

$$var_i = var_{i-1} - K_i * var_{i-1} \quad \text{[Equation 6]}$$

State Update Given Control Input:

$$x_i = x_{i-1} + u_i * T \quad \text{[Equation 7]}$$

Uncertainty Update Given Control Input:

$$var_i = var_{i-1} + var_u * T \quad \text{[Equation 8]}$$

Where the Kalman Weight Value is Specified by:

$$K_i = \frac{var_{i-1}}{(var_s + var_{i-1})} \quad \text{[Equation 5]}$$

2.6 Developing a Kalman Filter for a Practical SLAM Environment

Up to this point the formulas and mathematics involved in the development of the Kalman Filter have been intentionally kept very simple. We have now finished establishing the conceptual groundwork for the Kalman Filter and all that remains is to

incorporate the final two considerations examined at the end of the section 2.4 *Developing a Simple Filter*; we will now adapt the Kalman Filter for a multi-dimensional environment with multiple features. This will involve considerably more complex mathematics than we have seen so far, but the basic Kalman Filter steps will be exactly the same. We will continue to alternate between observing and moving. As before, observing will involve estimating feature locations which we will utilize to update and increase our confidence in the world state, and moving will provide new vantage points for the robot while decreasing confidence in the world state.

2.6.1 Adapted Notation for Multiple Dimensions

We will need to clarify how the meaning of certain notation has changed with our new more complicated environment. One very handy tip to keep in mind is that all uppercase notation refers to matrices, and all lowercase notation refers to vectors. No single valued variables will be considered (with the exception of i which will still refer to a time step).

Perhaps the most important thing to realize is that there must now be a distinction between feature locations and robot pose. In the previous 1-dimensional example there was only a single measured distance. Clearly, this is no longer the case. To properly update the state in a multidimensional environment we will need to keep track of all the feature locations. x_i will therefore refer to a vector containing both the robot pose and updated location estimates for the features. The first element of x_i, denoted xv_i, will correspond to the robot pose at time step i and the remaining elements will be feature location estimates. Note that a vehicle pose and a feature location will need to have

different forms—for example, a robot pose usually needs to contain both a location and a heading while a feature only needs a vector of coordinate values.

We need some way to refer to the feature location estimates. In our one-dimensional example, we used the variable z to refer to observations. We can simplify things for ourselves by modifying the meaning of z slightly to refer to actual estimates of feature locations and just forgo notation for observations. (Naming observations was useful to clarify the KF process, but is no longer necessary.) Additionally, we need to adapt the notation to distinguish between multiple features at a particular time step, i. To accommodate this, z_{ij} will refer to the j^{th} feature location estimate at time step i, and z_i will be the vector of all observations at time step i: $z_i = [z_{i0}, z_{i1} \ldots z_{in}]$. Note that z_{i0} is a vector specifying a physical location for feature 0.

Another very important change has to do with uncertainty. Previously we only needed to maintain a single variance variable *var*, but clearly this is no longer sufficient to model the uncertainty in the system. The uncertainty will now be contained in a covariance matrix P which we will need to continually update.

The movement vector u_i will retain its meaning as the predicted movement vector given control input at time step i, but this must now consist of both a velocity and a multidimensional orientation.

In the one dimensional example, K_i was a weight based on variances. Its function will be preserved but will now correspond to a matrix of values. Similarly, a single value to represent the uncertainty var_i will no longer be adequate; we will need an entire matrix P_i to represent uncertainty. K_i and P_i will be given more consideration shortly.

2.6.2 Adapted Movement Equations

The four KF update equations in this and the following section appear originally in the PhD thesis of Michael Csorba, with only the notation changed to be appropriate to this thesis.

In section 2.4 *Developing a Simple Filter* we began with simple formulas and derived a basic version of the Kalman Filter. To do so again here for a more complicated filter would be redundant because the same mathematical ideas are at play. The strategy we will employ in the current and the following section will be to present the appropriate Kalman Filter update equations at the outset and utilize the work we did in section 2.4 *Developing a Simple Filter* to understand each component of each equation in turn. Our work in *Developing a Simple Filter* will allow us to achieve a strong understanding of the KF as it applies to complex SLAM situations without delving too deeply into the mathematical nuances.

In the general form of the Kalman Filter used in practical SLAM environments, the updates necessitated by control input are handled by what are conventionally called "prediction equations" (Csorba, [1997]). (The idea is that when we move we need to "predict" how the state will change.) The sensor data is handled by "update equations", which update our state vector and uncertainty based on the data. This section will focus on the prediction equations.

The state prediction equation given actuator input is as follows:

$$x_i = F_i * x_{i-1} + G_i * u_i \qquad \text{[Equation 9]}$$

where F_i is called the state transition matrix and G_i is a matrix which translates control input into a predicted change in state (Csorba, [1997]). ([Equation 9] is the more complicated version of [Equation 7].) The role of F_i is to describe how we think the state

will change due to factors not associated with control input. One very nice convention in SLAM is that we get to assume that features will remain stationary. Except for the first row which correspond to changes in vehicle pose, F_i will therefore appear as a diagonal matrix with each diagonal entry containing identity matrices (otherwise F_i would change location of features which we know to be stationary). If our robot can have a nonzero velocity at time step i, then the state may change even with no actuator input, and the first column of F_i can account for this. To simplify things a little more, however, let's assume that the robot comes to a halt after each time step. In this case, the actuator input fully specifies the most likely new location of the robot. This means, F_i is just the identity matrix (or more accurately, a diagonal matrix whose diagonal entries are identity matrices) and can be completely disregarded. Thus, for our purposes, [Equation 9] simplifies to:

$$x_i = x_{i-1} + G_i * u_i \qquad \text{[Equation 10]}$$

The values in G_i will depend on the representation of the control input and will vary depending on the physical construction of the robot. Note that in [Equation 7], the time variable T was all that was needed to perform the task which G_i must do: translate control input to changes in state. Of course, with more control data, the translation process is more complex and an entire matrix (i.e. G_i) is needed. As just discussed, we will assume stationary features and therefore the only interesting entries of G_i will be in the first row. We will not concern ourselves further with the particulars of the entries in G_i as this is a housekeeping issue outside the scope of this essay. We can see that the only conceptual difference between [Equation 7] and [Equation 10] is that in [Equation 10] control input doesn't correspond directly with a change in state, so we need matrix G_i to

map control into a change of state. The premise of the two equations is the same; the best guess for our new state will be described exactly by our old pose and how we believe the actuators will change our pose.

Now let's take a look at the uncertainty prediction equation which is quite similar to [Equation 8]:

$$P_i = F_i * P_{i-1} * F_i^T + Q_i \qquad \text{[Equation 11]}$$

where Q_i is the covariance of the process noise (Csorba, [1997]), which will be elaborated upon shortly. As previously discussed, we may disregard F_i which will give the following equation:

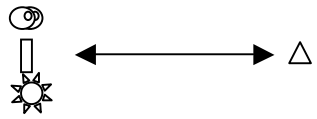
$$P_i = P_{i-1} + Q_i \qquad \text{[Equation 12]}$$

To understand this equation better, let's consider why the terms in this equation are matrices and not vectors. The matrix P_i is now taking the role of var_i. var_i was our way of measuring the uncertainty in our state prediction. There is now a unique uncertainty value associated with each of the features as well as the robot pose. It makes sense that we would need to keep track of multiple uncertainty values, but the reason why we need to keep track of a matrix of values rather than a vector might be less clear. (In particular, why don't we use a vector of n+1 uncertainty values, where n is the number of features?) The answer lies in what is meant by a "least squares" estimate. Before, we were concerned our confidence in a single relationship: relative position of the robot and the feature. Now the environment contains many more binary relationships and our confidence feature/robot and feature/feature pair applicable. In other words, we want to reduce the squared error in all relationships, and consequently we need to keep track of our confidence in all relationships.

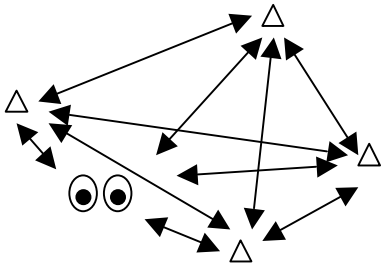
Let's take a moment to consider this a bit further. Suppose that according to the robot's most recent state prediction, the robot's orientation is off slightly. (In this case suppose this simply amounts to the predicted 360-degrees bearing of the robot.) All the feature estimates will be off as a result of this, but notice the feature estimates will all be off in a certain manner; in addition to sensor errors, the perceived locations of the features will all be rotated a particular angle with respect to the robot. Thus the error of any feature in the environment is related to the error of every other feature in the environment (Csorba, [1997]). To properly account for this fact and utilize all available information we must keep track of our confidence in how each feature plus the robot pose relates to every other feature and the robot pose. This is exactly the matrix, P_i.

In the case where there is only one error value to deal with, P_i simplifies to a 1 X 1 matrix with a single squared error entry—a variance, exactly as we had in section 2.4 *Developing a Simple Filter*. A visual comparison of the two situations appears below; this diagram should illuminate visually why P_i must be an n X n matrix with n = number of features (plus 1 for the pose):

In the 1-dimensional situation our confidence in this relationship was held in var_i:



Now confidence in *each* of these relationships needs to be represented in P_i:



Q_i must account for how moving will change our confidence each individual pair of features as well as feature-robot pairs. The entries of Q_i will depend on the distance features are away from the robot and other known particularities of the environment and/or state. As was the case with P_i, when there is only a single confidence value to keep track of Q_i will resolve to a single variance value. With these points in mind we can see that [Equation 8] is just a special case of [Equation 12] where there is only one uncertainty value. We have now described mathematically what happens to our state estimation and our confidence given actuator input according to the KF in our new complex environment.

Before we move on, we should notice that we are well on the way to achieving an important goal: simultaneity of mapping and localization. Notice that the matrix P_i allows us to update our confidence in the robot pose and feature locations simultaneously. Updating P_i amounts to updating confidence in the entire world state including both robot pose and feature locations. Similarly (as we will see very shortly), every value in P_i can be utilized to update the entire state given even a single sensor input. In other words, *truly simultaneous* localization and mapping can be performed every time new information becomes available. We should now understand how the Kalman Filter is a simultaneous solution to the SLAM problem.

2.6.3 Adapted Sensor Equations

The specification of the "prediction" equations in the previous subsection leaves only the sensor, or "update", equations, which are used to incorporate feature estimates based on sensor readings. These equations will be similar to Equations 4 and 6 in form, but it will be worthwhile to examine each in turn to understand exactly how the situation

has changed from the 1-dimensional example. As usual we will need to update the predicted state and our confidence in the current state using the new data. We begin with the state update equation:

$$x_i = x_{i-1} + K_i * v_s \qquad \text{[Equation 13]}$$

where v_s (also called the innovation) is an adjustment vector suggested by new sensor readings (Csorba). Let's look at v_s and K_i more closely. v_s is defined as follows:

$$v_s = x_s - H_i * x_{i-1} \qquad \text{[Equation 14]}$$

The purpose of H_i is to transform our previous state estimate into a representation used by the sensors. In other words, if the sensors had perceived the world state exactly as predicted by x_{i-1}, they would have returned this information in the form $H_i * x_{i-1}$. (Notice that the purpose of H_i is very similar to that of G_i. Once again we need not go into details of the matrix entries.) v_s, then, is essentially just the difference between the sensor estimate of the state, and the last cumulative estimate of the state. Substituting using [Equation 14] we have:

$$x_i = x_{i-1} + K_i * [x_s - H_i * x_{i-1}] \qquad \text{[Equation 15]}$$

This equation is so similar to the one-dimensional example that the equation as a whole can be understood conceptually from work we did to derive [Equation 4]. However, it behooves us to consider how K_i is constructed when the environment is multidimensional and contains multiple features—once we have done so, the remaining work in this section will be trivial. K_i is still based on the relative confidence of our sensors versus our previous estimate as it was in section 2.4 *Developing a Simple Filter*, but the variable K_i now refers to a matrix and is determined as follows:

$$K_i = P_{i-1} * H_i^T * S_i^{-1} \qquad \text{[Equation 16]}$$

We have already discussed how P_{i-1} and H_i are determined. S_i is the covariance of the innovation v_i and is determined as follows:

$$S_i = H_i * P_{i-1} * H_i^T + R_s \qquad \text{[Equation 17]}$$

where R_s is the covariance matrix for the sensor noise. The difference between R_i and S_i is subtle; it may be helpful to think of R_i as the tool we use to keep track of our confidence in our sensor readings and S_i as the tool we use to keep track of our confidence in the new state suggested by the readings given the fact that our sensor readings differ from what we would have otherwise predicted. Note that both R_i and S_i are matrices, with dimensions one plus the number of features. In section 2.4 *Developing a Simple Filter*, sensor noise could be represented with just a single variable, var_s. To understand why S_i and R_i are matrices, see the arguments we used when we defined P_i in the previous section. Combining [Equation 16] and [Equation 17] we see the Kalman matrix constructed in this manner:

$$K_i = P_{i-1} * H_i^T * [H_i * P_{i-1} * H_i^T + R_s]^{-1} \qquad \text{[Equation 18]}$$

Remember that H_i and H_i^T just do clerical work of relating sensor data to the world state and vice versa. If we ignore H_i and H_i^T, we can see clearly how [Equation 18] is analogous to a slightly rewritten version of [Equation 5]: $K_i = var_{i-1} * [var_{i-1} + var_s]^{-1}$. (Remember we said that P_{i-1} is analogous to var_{i-1} and R_s is analogous to var_s.) Of course, the symbol -1 indicates inverse rather than power in [Equation 18], but the conceptual function of the symbol is the same: the magnitude of the values in K_i will depend on the previous uncertainty relative to the combined values of the previous uncertainty and sensor uncertainty.

The uncertainty update equation will complete our discussion of the KF update formulas. After we have updated our state using [Equation 13] we can update P_i as follows:

$$P_i = P_{i-1} - K_i * S_i * K_i^T \qquad \text{[Equation 19]}$$

We have already defined all the terms in this formula. Intuitively, the purpose of this equation is the same as the purpose of [Equation 6]. We want to update our confidence based on the combination of the previous confidence and the sensor confidence. Just as in [Equation 6], the new confidence will always be higher (i.e. with lower covariance entries) than previously. Also, the new confidence will proportionally improve most when the sensor confidence is similar to the accumulated confidence.

We have now considered prediction and update equations for both state and uncertainty, and in so doing have specified the Kalman Filter for a multidimensional environment with multiple features. We should take a moment here to appreciate what we have accomplished in the last two sections. We have explained mathematically how a robot may alternatively move and map an unknown environment with unlimited features and multiple dimensions. We have described how to use all available knowledge in a realistic SLAM environment to maintain an accurate confidence level in each aspect of the state, and how to use that confidence to make the best possible state estimation given new data. Additionally, we have taken care to make each of the four Kalman equations as analogous as possible to the equations we rigorously developed in sections 2.4 *Developing a Simple Filter* and 2.5 *Developing A Kalman Filter For a Moving Robot*. We have also taken care to properly explain the presence of any conceptually new aspects of the equations which we did not see in those sections. In short, we have specified the

steps involved in the optimal least squares SLAM solution, and further we have explained the purpose of each of these steps and described how they work in a careful and intuitive manner.

Of course, there is a lot more we could says about the KF, but the goal of this thesis is not to enumerate every detail of the Kalman Filter but rather to provide an explanation of a few SLAM techniques in a manner that does not require a strong background in robotics to understand. While understanding the Kalman Filter was a goal of this thesis in and of itself, the KF will also serve as a stepping stone to other, more state-of-the-art techniques. After a brief discussion of the Extended Kalman Filter, our discussion of the KF will draw to a close. For reference, the equations which comprise the KF solution presented in the last two sections are repeated here, with their original equation numbers:

Kalman Filter equations for practical SLAM environments:

State Update Given Actuator Input:

$$x_i = x_{i-1} + G_i * u_i \quad \text{[Equation 10]}$$

Uncertainty Update Given Actuator Input:

$$P_i = P_{i-1} + Q_i \quad \text{[Equation 12]}$$

State Update Given Sensor Readings:

$$x_i = x_{i-1} + K_i * v_s \quad \text{[Equation 13]}$$

Uncertainty Update Sensor Readings:

$$P_i = P_{i-1} - K_i * S_i * K_i^T \quad \text{[Equation 19]}$$

Where:

$$v_s = x_s - H_i * x_{i-1} \quad \text{[Equation 14]}$$

$$K_i = P_{i-1} * H_i^T * S_i^{-1} \quad \text{[Equation 16]}$$

$$S_i = H_i * P_{i-1} * H_i^T + R_s \quad \text{[Equation 17]}$$

2.7 The Extended Kalman Filter

The Extended Kalman Filter (EKF) is very similar to the Kalman Filter. Equations 13 and 19 are utilized unchanged in the EKF, and Equations 10 and 12 need to be modified only slightly for use in the EKF (Csorba [1997]). An in-depth discussion of the new equations would require the description of complex mathematical concepts and would not contribute to our understanding of SLAM a significant way. However, the Extended Kalman Filter deserves a little consideration here as it is more commonly employed than the basic KF in practical SLAM situations. The reason for the popularity of the EKF is that it can handle nonlinear process models (see section 2.3 *Concepts: White Noise, Gaussian Curves, and Linear Processes*), and in most real world SLAM situations there will be some non-linear aspect we wish to account for.

The EKF assumes that a function, f, describes how the state changes from one time step to the next time step. The EKF further assumes f can be computed to generate a new state estimate given a state, time index, and control input. (In a SLAM context we are only worried how the robot pose changes, not the entire state, because we are still assuming features are stationary.) Thus we may rewrite Equation 10 as

$$x_i = f[x_{i-1}, u_i, i-1] \qquad \text{[Equation 10b]}$$

(Durant Whyte [2002]). The EKF does not assume, however, that f can be described by a discrete equation. This causes a problem when we want to update the covariance matrix—we can't describe how our certainty changes when we don't expect the state to change in a discrete way.

To solve this problem, the EKF uses the Jacobian of f (an approximation of how the state will change) to approximate how we should expect our certainty to change. In

the interest of brevity and to steering our discussion toward SLAM topics as opposed to mathematics topics, we will forgo an in-depth discussion of the Jacobian. Put intuitively, the Jacobian of f, Jf, is a matrix which describes how the state would change after time = i given that the state begins to change in a linear manner at exactly time = I (Csorba [1997]). For example, suppose a robot's speed is changing according to some complex function f, and at time = i the robot is slowing down at a rate of 1 meter per second or m/s. We might expect that at time = i + 1 the robot's acceleration will be different than 1 m/s, but the EKF would assume a constant deceleration for the purposes of calculating the covariance matrix. The EKF update equation for uncertainty is:

$$P_i = Jf * P_{i-1} * Jf^T + Q_i \qquad \text{[Equation 12b]}$$

(Durant Whyte [2002]).

Note that if we maintain our assumption that the robot stops between each set of observations, the Extended Kalman Filter is of no use at all; the Jacobian can be discounted at each step because all features and the robot are stationary. We have considered the EKF only because it is the version of the KF most frequently used in SLAM techniques and frequent references are made to the EKF in SLAM literature. It is important to know a little about the EKF simply to improve our understanding what most SLAM techniques to date have focused on.

2.8 Limitations of Kalman Filter Techniques

The Kalman Filter, as we have argued, is a very good SLAM tool. One might wonder how we can improve on an optimal least-squares approach like the KF. Of course, given a particular physical environment and a particular robot there will be numerous heuristics which can be combined with the KF to boost performance. However,

it might seem from the arguments we have made so far that the KF is in general the best starting place for any SLAM problem. If the sheer number of variations developed by researchers is any indication of the KF's merit, then the KF is a good place to start indeed; the KF is, as we have said, the basis for (by far) the majority of SLAM approaches (Durrant-Whyte, [2002]). On the other hand, many recent solutions have come at SLAM from other directions. It is time to consider how the Kalman Filter might not be the best solution in many situations after all.

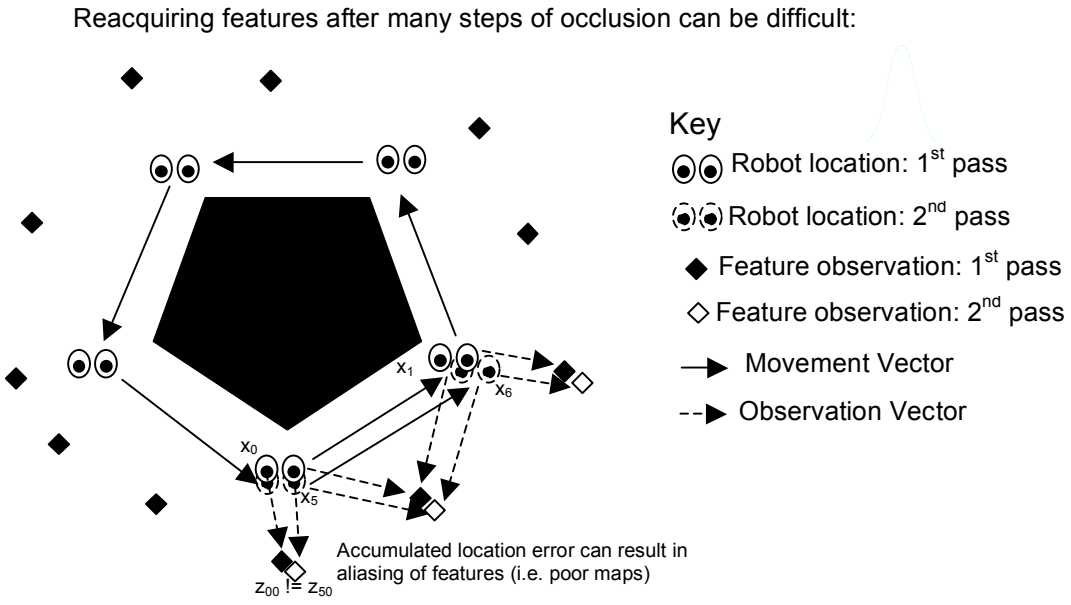
KF approaches work extremely well for environments in which there are a particular number of easily trackable features (M, T, K, & W, [2002]). This is a result of the time complexity of the KF. Remember that the covariance (uncertainty) matrix updated by the KF, P_i, is quite large because it needs to relate each feature to every other feature. We must update each of these values for every iteration of the filter irrespective of how many features are actually located. Therefore, the KF has a complexity of $O(m^2)$ for each iteration, where m is the number of features. The practical result of this is that, in order to work in real time, the number of features must be somewhere in the ballpark of a few hundred (M, T, K, & W, [2002]). The case can be made that the KF is doing far too much work. For example, there is no guarantee that in practice that uncertainty values maintained by the KF will accurately reflect the actual error in the sensors or in the state prediction (Castellanos, Neira, Tardos, [2004]). It seems that maintaining covariances is computationally expensive and also not always as useful in practice as it is in theory. For these reasons, some have decided that more time should be spent on other things, for example utilizing additional features (M, T, K, & W [2002]).

Most real world environments have many more than a few hundred theoretically trackable features in them. In these environments we see the flaw in the claim that the KF utilizes all available information; the KF certainly could take every available feature into account, but the price might be a very long wait or the need to hand over the data to faster hardware to be processed. In many cases, the only alternative is to ignore the vast majority of features and consequently the vast majority of available data. It practice it is sometime better to take every feature into account, and make up the time by using an algorithm which represents uncertainty without a single large covariance matrix for the feature estimates.

KF approaches suffer from another draw back in the opposite situation. Many environments have few if any trackable features. The problem of feature tracking is a complicated one, and we can never guarantee that a robot will be able to find a feature again after it moves. Additionally, if there are repeating patterns in the environment the wrong feature might be relocated or one feature might be mistaken for another. Suppose our robot is placed in such an environment equipped with a laser range finder. The claim that the KF utilizes all available information is not quite accurate once again; using the laser range finder alone may be more useful than attempting to follow a few poor features around with a KF (Eliazar and Parr, [2004]).

One final limitation of the KF which is particularly applicable to this thesis manifests itself when the robot leaves an area in the environment and returns to it again later. Even if the old features can be relocated, the robot may have difficulty realizing it has already seen them and count them as new features. Intuitively, it seems that returning to a familiar environment should improve the robot's state rather than make it worse, but

the KF often requires special "loop closing" heuristics to handle the situation properly (Eliazar and Parr, [2004]). The robot's ability to handle previous locations is known as its loop closing capability. Loop closing is one of the most difficult problems facing SLAM researchers today (H, B, F, & T [2003]). To see why loop closing is a problem, imagine a robot attempting to move back to a previous position after maneuvering around some obstacle. The accumulated errors that resulted from prolonged occlusion of familiar features might be enough to prevent the robot from recognizing the features it previously used. The problem of loop closing is illustrated below:



We will now take a look at two alternate SLAM methods which excel in situations the KF is unsuited for.

3 FastSLAM

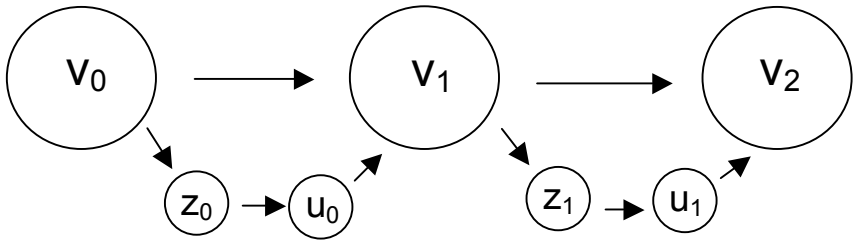
We ended our discussion of the Kalman Filter by noticing that KF based techniques have limitations which come into play in certain situations. In particular, we noticed that in environments where millions of features are available, relatively few can be utilized by the KF in a reasonable amount of time. This is the particular limitation of the KF that FastSLAM attempts to improve upon. FastSLAM was developed by Montemerlo, Trun, Koller, and Wegbreit who published a definitive paper on the algorithm in 2002 (M, T, K, & W, [2002]). FastSLAM breaks up the problem of localizing and mapping into many separate problems using conditional independence properties of the SLAM model (we will discuss exactly what is meant by this shortly). In brief, FastSLAM is capable of taking far more raw sensor data into account by concentrating on the most important relationships between elements in the environment. The result is a SLAM solution that performs much better than the KF in many environments. For example, environments with many random patterns, shapes, and textures often work quite well for the FastSLAM algorithm.

3.1 A Bayesian Approach to SLAM

FastSLAM suggests presenting the SLAM problem from a Bayesian point of view (M, T, K, & W, [2002]). A Bayesian model of the world is one in which there are certain "random variables"—variables which take on different values with different probabilities depending on the values returned by other variables. The result is a tree of random variables which depend on one another, and specify the likelihood of different world states. This network is called a Bayesian network. One of the main reasons for representing processes with a Bayesian network is so that conditional independences can

be taken advantage of (to be explained shortly). It can be argued that to some extent we have been using a Bayesian model all along. For example, we could use the following diagram to illustrate the SLAM process as we have been thinking of it:

Simplified Bayesian Network for SLAM:

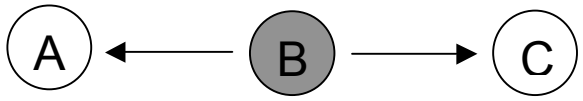


Notice that V refers to an actual vehicle location, as opposed to xv_i which corresponds to a vehicle position estimate. The arrows indicate direct causal dependences—the value of a random variable directly influences the probabilities associated each of its children. v_1, for instance, depends on both the previous vehicle position and the last control input. Variables in the diagram can be dependent on other variables even if there are no arrows directly connecting them. For example, suppose we did not know the value of v_1, and we want to know the value of v_2. In this case, we would know more about v_2 if we at lest knew v_0, the original vehicle position (although not as much as if we knew v_1). Hence, v_2 is dependent on v_0. On the other hand, if we knew the value of v_1, v_0 would be entirely irrelevant to the value of v_2, and we say v_2 is conditionally independent of v_0 given v_1.

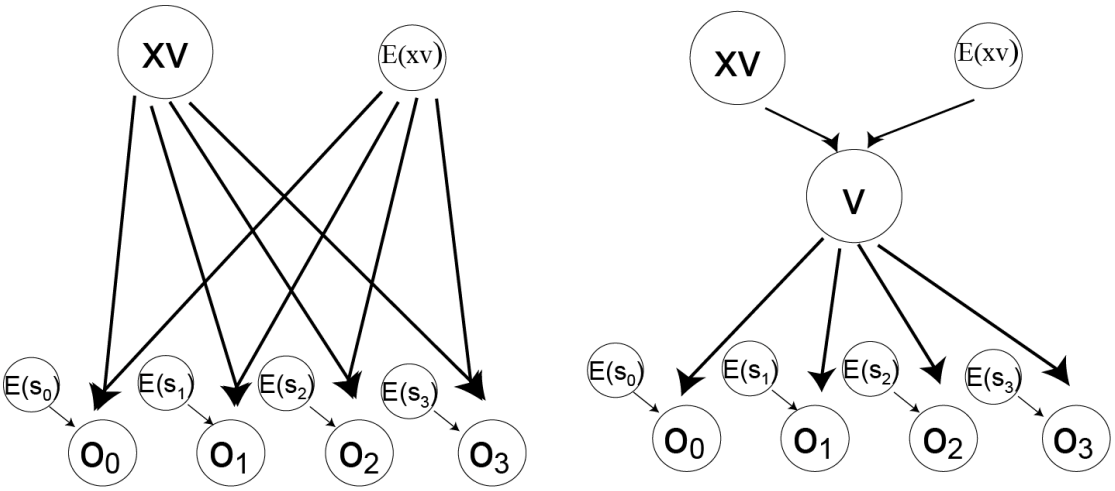
Conditional independence is what allowed the Kalman Filter to make optimal guesses while only updating a state prediction and uncertainty values; all previous sensor readings, state predictions, and control inputs are irrelevant given the most recent state prediction and uncertainty values. The rules for conditional independence in a Bayesian

network are defined by a set of d-separation rules. Two variables are said to be independent of each other if every path between them is d-separated. Only one d-separation rule is pertinent to our discussion and is illustrated below (Scharstein, [2005], Huang [2006]):

A and C are d-separated given the true value of B:



Let's consider a Bayesian network which relates the robot position to the observations made by the sensors. We would expect none of the observations to be conditionally independent of any other feature given a pose estimate. We noticed this when we defined P_i in section 2.6.2 *Adapted Movement Equations*; errors in the pose estimate result in an inextricable relationship between the sensor based feature estimates. We can use the following diagrams to help us understand the pertinent conditional dependence at play in SLAM. In the figure below, the Bayesian network on the left illustrates how observations taken by the robot are affected by pose-estimate and vehicle-position-error random variables. The Bayesian network on the right represents a similar network that takes into account the actual vehicle pose v, which is defined by xv_i and $E(xv_i)$:



Key:
xv = estimated robot pose
v = actual robot pose
E(xv) = error in estimated robot pose
$E(s_i)$ = error in ith sensor reading
o_i = observation based on ith sensor reading

Clearly we can find no conditional independence relationships in the Bayesian network without incorporating v; even if we pick an estimate for xv_i to create a d-separation, the observations are still indirectly dependant on one another through $E(xv_i)$. Notice, however, in the network on the right the individual landmark estimates are conditionally independent of one another given the actual robot location by the d-separation rule previously illustrated (M, T, K, & W, [2002]). This should make good intuitive sense. Remember when we discussed the need for a matrix of uncertainty values, P_i, we argued that if the predicted robot orientation was slightly off, each of the features estimates would be slightly rotated and therefore be related to one another. In fact, the only way feature estimates should ever be related is through error in the robot pose. If we know the robot position exactly, there should be no predictable relationship between the feature observations.

Of course, a very major point in SLAM is that we do *not* know the exact robot pose, but the insight of conditional independence of features given the pose is enough to motivate FastSLAM, which handles each feature separately. The thinking is, perhaps the relationships between features aren't worth keeping track of if we can make a good enough estimate (or estimates) of the random variable that, when known exactly, d-separates all feature estimates from one another. This variable is, as just described, the true robot pose.

3.2 The FastSLAM Algorithm

Let's take a look at how the conditional independences described above allow us to divide up the SLAM problem. We will need the use of a probability operator, *p*. Given any input, *p* returns the probability of that input. For example, suppose our robot's one-dimensional position is specified by the variable x. Then $p(x)$ will approximately be a Gaussian curve, just as we saw in section 2.4 *Developing a Simple Filter*. If x is a multidimensional vehicle pose and feature estimates, $p(x)$ defines the likelihood of all possible world states. | is the "given" operator. $p(x \mid \{u_0, u_1, \ldots u_i\}, \{z_0, z_1, \ldots z_i\})$ therefore describes likelihood of any world state given all sensor and movement data taken by the robot at any time i, just as P_i and x_i did in the more complex version of the Kalman Filter. (For compactness, we will say $U^i = \{u_0, u_1, \ldots u_i\}$ and $Z^i = \{z_0, z_1, \ldots z_i\}$.) One of the benefits of the Kalaman Filter was that no matter how large i became, P_i and x_i remained the same size as long as we kept using the same features. The unbounded nature of $p(x \mid U^i, Z^i)$ as i increases is something we will need to deal with later. Notice that x_i referred to a specific world state, while x refers to all world states. x can, as would be expected, broken down into vehicle pose, v, and feature locations $p_0, p_1, \ldots p_m$. Like x,

these variables represent all possible locations and poses. We can break down $p(x \mid U^i,Z^i)$ as follows.

$$p(x \mid U^i,Z^i) = p(v,p_0,p_1,\ldots p_m \mid U^i,Z^i)$$

We will now use some probability arithmetic to simplify the SLAM problem. Suppose we have two random variables which are not related, A and B. We could say, $p(A, B) = p(A) * p(B)$. If the value of A depended on the value of B, this identity would not be valid. (To see why this is true, suppose A and B were either true (1) or false (0) each with a probability of .5, that A always equaled B, and somehow we discovered that A was 1. The left side of the expression would be 1 and the right would be .5.) If A does depend on B, the proper factorization is $p(A, B) = p(A) * p(B|A)$. We know our feature predictions depend on the state prediction which means we can break

$p(v,p_0,p_1,\ldots p_m \mid U^i,Z^i)$ down as follows:

$$p(v,p_0,p_1,\ldots p_m \mid U^i,Z^i) = p(v \mid U^i,Z^i) * p(p_0,p_1,\ldots p_m \mid U^i,Z^i,v)$$

We can now see how the independence relationship we discovered in section 3.1 *A Bayesian Approach to SLAM* might help us. Because feature observations are conditionally independent of one another given the robot pose, and we (hopefully) have a good approximation of the robot pose, it seems as though we might break $p(p_0,p_1,\ldots p_m \mid U^i,Z^i,v)$ up into separate probability statements. Of course, all we can really have are estimates for v, but if we have acurate enough estimate (or as we will discuss shortly, enough estimates) we can make use of the following factorization:

$$p(v \mid U^i,Z^i) * p(p_0,p_1,\ldots p_m \mid U^i,Z^i,v)$$

$$= p(v \mid U^i, Z^i) * \; p(p_0 \mid U^i, Z^i, v) * p(p_1 \mid U^i, Z^i, v) * \ldots p(p_m \mid U^i, Z^i, v)$$

To summarize, we have:

$$p(x \mid U^i, Z^i) = p(v \mid U^i, Z^i) * \; \prod_m p(p_m \mid U^i, Z^i, v) \qquad \text{[Equation 20]}$$

(M, T, K, & W, 2002).

Let's take a moment to think about exactly what this factorization accomplishes. We have broken the SLAM problem into m+1 separate problems, and none of the m feature estimation problems are related to one another. It seems that this might help us solve the cause of the polynomial complexity of the KF. (Of course, we need to develop a way to solve each of these m+1 problems in a timely manner, but intuitively this should seem possible.) The only price we pay for utilizing this factorization is a possible decrease in accuracy due to ignoring correlations in feature location errors due to inaccuracies in the predicted robot pose. An interesting result of all of this is that if we were ever were to have 100% confidence in our robot pose, our conditional independence assertion would be completely applicable and we could disregard correspondences between features. In this case, solving the factorized version of the SLAM problem in [Equation 20] would yield an optimal guess for feature locations far faster than a KF approach.

Furthermore, if we knew the true robot pose to be one of only a finite number n, it would still be possible to utilize the factorization in [Equation 20] as long as we knew the likelihood of each of the n possible poses. For example, we could make a separate map for each robot pose (each time disregarding feature error correlations) and then weight each map accordingly. This would properly account for the correlation between observations, and therefore generate an optimal state estimate. Additionally, this would

likely have a better time complexity than a KF approach, depending on the number of features and the number of pose possibilities. In practice, there will be few if any cases where we can narrow the possible poses to a finite number. However, if we keep track of many possible poses and weight the likelihood of each appropriately, we might be able to approximate the optimal least squares guess sufficiently for all practical purposes while doing far less work than a KF. This is exactly the strategy employed by FastSLAM (M, T, K, & W, [2002]).

3.3 FastSLAM and Particles

The FastSLAM algorithm keeps track of many possible paths simultaneously (M, T, K, & W, [2002]). Notice that the KF does not even keep track of a single path, but rather updates a single robot pose—the latest step in the robot path. FastSLAM records paths so that when the algorithm terminates there will be a record of where the robot has been and where it stopped along the way; FastSLAM does not use old poses in the paths for any calculations, so old pose estimates are essentially forgotten during execution (M, T, K, & W, [2002]). As the full robot paths are not utilized and therefore unimportant to the SLAM problem as we have defined it, we will present a slightly adapted FastSLAM algorithm which updates pose estimates rather than grows robot paths. This will simplify notation and generally make the formulas look more familiar to ones we have seen already in this thesis.

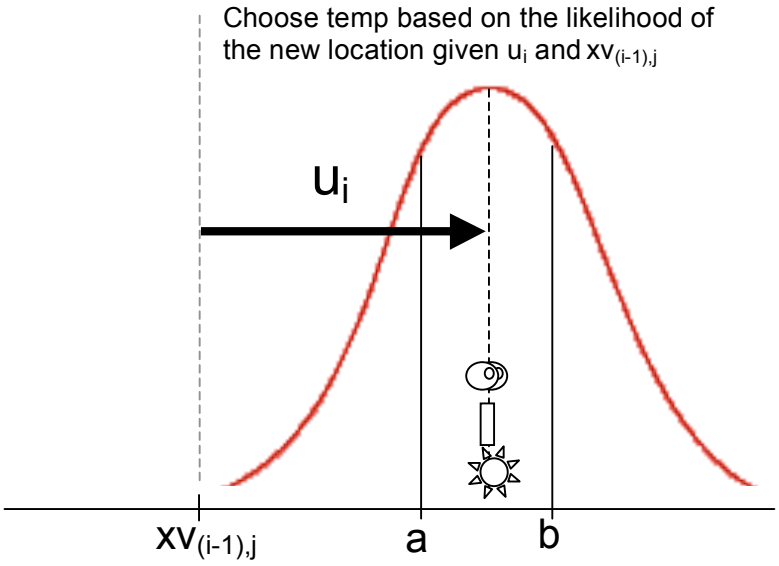
Maintaining multiple pose estimates at each time index necessitates a complication of our notation for robot pose. Previously, xv_i referred to the vehicle pose at time i. Now xv_i will refer to the set of robot poses we have maintained at time i. xv_{ij} will

refer to the j^{th} pose estimate at time i. Therefore we have, $xv_i = \{x_{i0}, x_{i1}, \ldots x_{iM}\}$, and each estimate x_{ij} is called a particle. Now we need to specify how to update xv_i.

3.4 Pose Estimation

The particles in xv_i will be updated in a probabilistic way—that is to say, there will be a certain amount of randomness involved in updating the particles. We don't want our particles to be homogeneous, and we also want a greater percentage of "good" estimates to survive. This is what updating xv_i probabilistically allows. FastSLAM is only an approximation of the least squares estimate, so we will not invalidate any optimality claims by doing this.

We will now discuss how to maintain the particle set. Suppose we have a set of pose estimates xv_{i-1}, and we have a control input u_i. The particle set is not updated until we make some feature location estimates from observations in our new location. Instead, we create, xv_t, a temporary set of particles based on u_i and the M particles in xv_{i-1}. Both the number, M, of particles we maintain in the filter, and the number we generate for xv_t will depend on the situation. Testing of FastSLAM in practical situations has shown that increasing the number of features can reduce the number of necessary particles (M, T, K, & W, [2002]). xv_t will be chosen probabilistically by sampling from xv_{i-1}. For example, to generate a particle for xv_t we start with a particle from xv_{i-1}, say $xv_{(i-1),j}$. We then use the probability density specified by $xv_{(i-1),j} + u_i$ to choose temp, a new particle for xv_t. This process is illustrated below:



$$p(a < temp < b) = \int_a^b p(xv_{(i-1),j} + u_i)$$

This process is repeated, choosing "better" particles from xv_{i-1} (i.e. robot poses we have the highest confidence in) more frequently, until there are N particles in xv_t with N >= M. (By increasing N and/or M, we can more closely approximate the optimal least squares solution, but doing so also makes the FastSLAM algorithm take longer. N and M should be determined experimentally.) Once we have xv_t we will need a method of selecting the best particles from xv_t to carry over into xv_i. This is where the feature observations come in. Each particle in xv_t, $xv_{tk,}$ is given a particular normalized weight, w_{tk}. We base the weight on the confidence increase in the particle given the new observations; in general we want to keep the particles which the new observations support and throw out the particles which are contradicted. Put mathematically:

$$w_{tk} = \frac{p(xv_{tk} \mid Z^i, U^i)}{p(xv_{tk} \mid Z^{i-1}, U^i)} \qquad \text{[Equation 21]}$$

Now we can build xv_i by probabilistically selecting M particles from xv_t. Let's take a moment to see what sort of qualities we might expect from a set of particles

maintained in this fashion. We choose xv_t so that most of the particles/poses are close to what we would expect them to be given our previous particles and control input. Then the temporary particle set, xv_t, is whittled down, and particles which do not reflect the new sensor readings as well as we expected them to tend to drop away. In short, we maintain a set of poses/particles which tend to cluster around the optimal estimate of the robot pose given all sensor data and control input—i.e. the estimate which would be made by the KF. In other words, xv_i represents an approximation of $p(v \mid Z^i, U^i)$, and the error in this approximation approaches zero as M, the number of particles approaches infinity (M, T, K, & W).

3.5 Feature Estimation

It may be surprising to know that we are almost finished detailing the operation of the FastSLAM algorithm. We did, after all, break the SLAM problem into m + 1 separate problems, and we have solved only one of these. The remaining m problems, however, all refer to feature locations estimates and can therefore be handled in the same way. All we need is some way of estimating the now uncorrelated landmark values very quickly. We have, quite conveniently, already developed such a method in this thesis, in section 2.4 *Developing a Simple Filter*. Ironically, it was the limitations of the KF which motivated consideration of FastSLAM in this thesis, and now we will be using multiple simple Kalman Filters in the implementation of FastSLAM.

We need not dwell on the Kalman Filter implementation in the FastSLAM algorithm as we have already discussed Kalman Filters in detail, but some consideration of complexity is in order. We will need to have 1 Kalman Filter per feature, per particle, for a total of M * m filters. Each filter will be conditioned on its respective particle pose,

and will need to estimate one feature position. In two dimensions, we can uniquely identify a feature location with a range and bearing—just two variables as opposed to the m variables we had in section 2.6 *Developing A Kalman Filter for a Practical SLAM Environment*. The covariance matrix for each filter, then, will be 2 X 2, and contain only four values. (We cannot deny a correlation between error in range and error in bearing, and consequently we will need all four values. This will take some time, but does not change the time complexity.) This means that each filter will be only slightly more complicated than the one we developed in 2.4 *Developing a Simple Filter*, and the filter update can still iterate in constant time.

3.6 Loop Closing with FastSLAM

Loop closing, to a certain extent, is handled naturally by FastSLAM. The overall increase in mapping accuracy made possible by utilizing this quantity of landmarks can often prevent problems with loop closing (M, T, K, & W, [2002]). It is worth while mentioning that FastSLAM's system of weighted particles has inspired researchers (such as Stachniss and Burgard) to develop active loop closing techniques. We will briefly discuss how such techniques work.

Earlier we said that each particle in the temporary set is given a weight based on new observations. When the robot enters a new area, the robot has no idea what the area should look like, so the new observations are not likely to contradict any of the particles more than others. Consequently, the weights of all the particles are likely to be similar (Stachniss, Burgard [2004]). When the robot reenters an old area, on the other hand, one of two things can happen. The first possibility is that some particles will make good estimates while others won't. In this case, the weights will be diverse and bad particles

are more likely to be weeded out. The other possibility is that all the particles will make bad estimates and the loop will be closed poorly even by the best particle. This is second possibility is more likely the case when the robot has been away for a long time.

From these arguments it seems that reentering old areas a soon as possible is beneficial (Stachniss, Burgard [2004]). Indeed, experimental testing has confirmed this idea. In 2004 Stachniss and Burgard attempted mapping hallways first allowing the robot to revisit an old area half way through. They then repeated the experiments without allowing revisitation. The maps resulting from the "revisitation" routes resulted in much more accurate maps (Stachniss, Burgard [2004]). Stachniss and Burgard also developed some heuristics which a robot can use to decide what area to explore next, but to go into them here would mean staying too far from the main drive of this thesis.

3.7 FastSLAM as a Practical Real-Time Solution

The computational complexity of FastSLAM as we have described it (for a single observation) is O(m * M), or the number of feature multiplied by the number of particles. The O(m * M) run time is a result of copying old maps for the new set of particles. If, instead of copying each map, we change the appropriate feature in each particle, we can lower the complexity significantly. Sorting through the m features can be done in log(m) time if we use a tree structure to store the maps. Storing features in this way results in a total complexity of O(M * log(m)). This is in contrast to KF approaches which run in $O(m^2)$ time given even a single observation.

Experimental testing has shown that in general, a few particles will suffice irregardless of the number of features, so the complexity of FastSLAM tends to be far better than the complexity of the KF (M, T, K, & W, [2002]). FastSLAM has been tested

in physical environments with as many as 50,000 landmarks, which were mapped accurately with as few as 250 particles (M, T, K, & W, [2002]).

If we used a Kalman Filter to keep track of the same number of features, it would need to make more than 2.5 billion covariance updates given even a single observation of a single landmark. Clearly FastSLAM represents a vast improvement over the KF given an environment where tens of thousands of features need to be mapped, particularly when we consider the improved loop closing potential of FastSLAM.

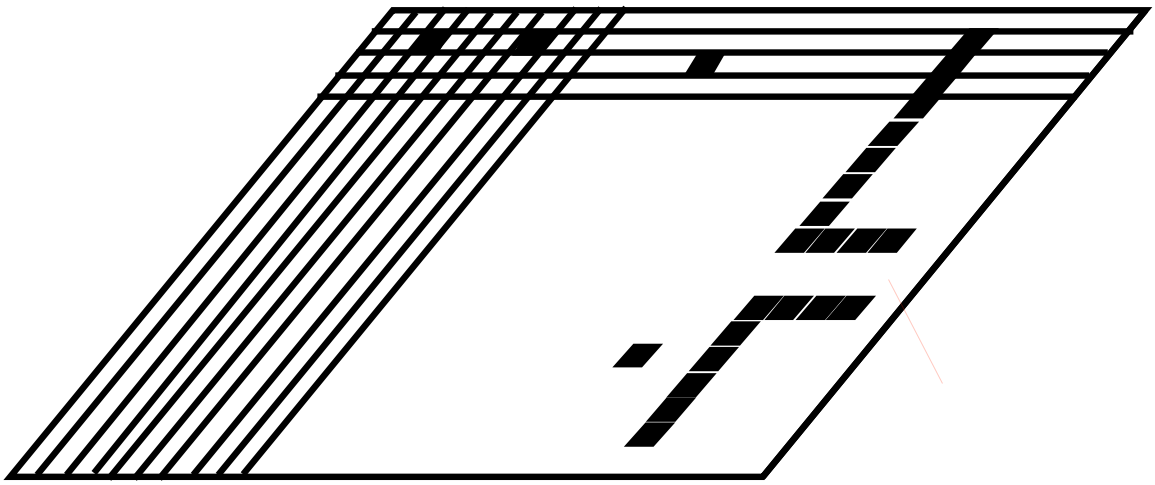
To summarize, FastSLAM is based on the conditional independence of features given an accurate robot pose. The SLAM problem can be broken into m + 1 separate estimation problems if we can account for errors in the robot pose. We approximate the optimal state estimate by keeping track of multiple pose estimates (i.e. particles) and weighting the map based on each particle appropriately. The particles are maintained by probabilistically sampling from the previous poses, and probabilistically removing poses which do not reflect new sensor data. The result is an approximation of the optimal mapping algorithm which executes in real time with tens of thousands of features.

4 DP-SLAM

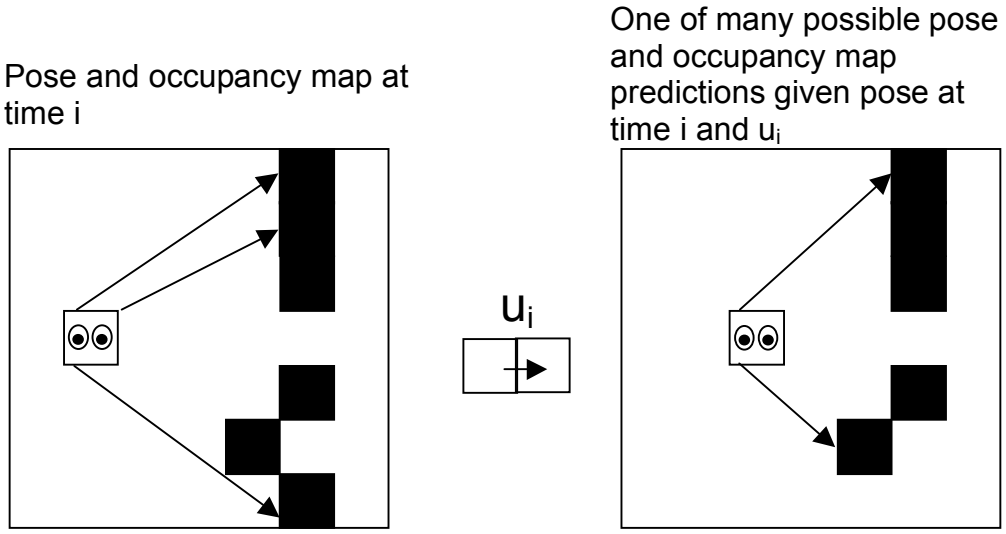
We have discussed how FastSLAM is a good solution to the SLAM problem when there are thousands of features that can be tracked accurately. The availability of good features is something we have, until now, taken for granted. Tracking features is actually very difficult, particularly in environments with monochromatic areas or repeating patterns. Suppose there is an environment which needs to be mapped, but the robot can't find enough features to effectively employ FastSLAM or even a Kalman Filter. Further suppose that we could very quickly sense the distance to nearby occupied points using a laser range finder. The map generated by the laser has no features, but is rather an occupancy grid. (For an example of an occupancy grid see the next diagram.) The robot cannot use the range finder to relocate individual points in the grid. However, with enough data, the robot might not need to worry about reacquiring features in the first place. For example, if there is a distinctly contoured object in the environment, the robot might be able to align entire occupancy maps by matching up the contour of that object. DP-SLAM attacks SLAM from this occupancy grid approach, while simultaneously utilizing the conditional independence insight we found in our discussion of FastSLAM (Eliazar and Par [2004]).

4.1 Features as indistinct points

An occupancy grid is a two dimensional array with each cell being either empty or occupied. For example, the following diagram could be a low resolution occupancy map of a room or hallway (depicted from an angle with only some of the grid lines filled in). The series of filled cells on the right might be a doorway:



One of the steps in the FastSLAM algorithm was to generate a new pose prediction and a corresponding map prediction. The new map prediction was based on the parent pose and the control input. We cannot predict how features will move with respect to the robot anymore because we aren't dealing with features. However, we can make a prediction as to how the occupancy grid will change. For example, consider the following diagram which illustrates a (max likelihood) occupancy grid prediction based on a move of one cell to the right:



In the context of DP-SLAM, an occupancy grid is a two dimensional structure. Although it is theoretically possible to use a laser range finder to map multiple height levels of a robot's environment, this is outside the scope of DP-SLAM. This is a significant limitation of DP-SLAM. Mapping on uneven terrain, for example, would be almost impossible from a DP-SLAM standpoint. As we will see later, DP-SLAM is still quite useful in many (especially indoor) environments.

4.2 DP-SLAM: an Extension of FastSLAM

Now that we have an idea of what sort of data DP-SLAM is dealing with, let's take a look at how the algorithm actually functions. As we described in our discussion of FastSLAM, without knowing the exact position of the robot, all observations are correlated to one another. (This includes the occupancy observations we are concerned with now.) Once again, we can avoid relating each observation to every other observation by maintaining multiple weighted poses; with enough poses we can assume observations are independent of one another without deviating significantly from the optimal estimate. Ironically, the same conditional independence insight that allowed us to deal with thousands of features in the FastSLAM algorithm also allows us to navigate with out any features at all with DP-SLAM.

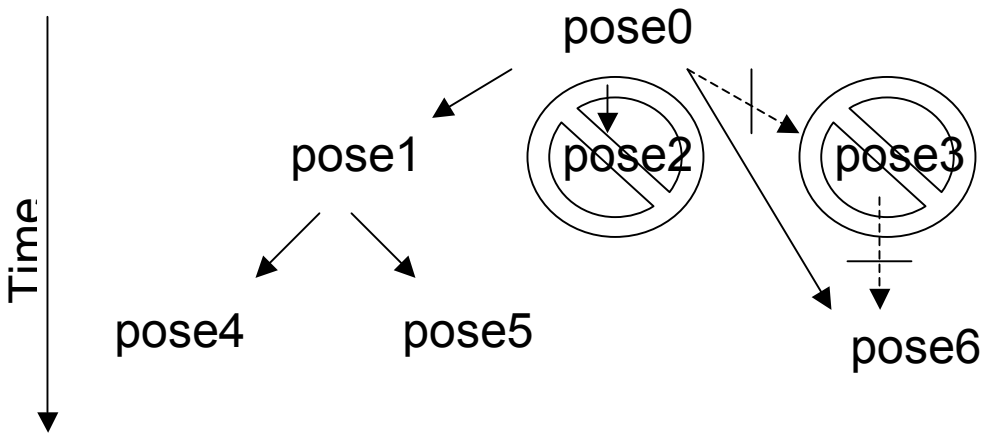
Just as we had with FastSLAM we will need to generate new particles (poses) by applying probabilistically generated movement vectors to old poses. Like FastSLAM, DP-SLAM involves choosing poses to keep based on new sensor data. The estimation and selection processes are so similar to those of FastSLAM that no more needs to be said here.

4.3 A Tree of Particles

We claimed earlier that we would keep track of multiple robot poses each with a unique occupancy grid. This claim is true, but also somewhat misleading; there will be a unique occupancy grid which corresponds to each particle, but we will not keep track of those grids explicitly. In a realistic environment we will want large grids to more finely describe the position of objects and structures. Creating an entire new occupancy grid for each new particle is generally inefficient. Remember that when we create a new occupancy grid, it is probabilistically based on a parent grid and a movement vector. In particular, it should only differ from the parent grid in places which can be observed with the laser range finder from the new position (Eliazar and Parr [2004]). (We will refer to the size of the area swept out by the laser as A.) One obvious solution is to only store discrepancies between the child occupancy grid and the parent occupancy grid. Assuming that the laser only provides data for a small portion of the entire grid, storing only the discrepancies between the parent and child node would save a lot of time and space. Consequently, in DP-SLAM we will maintain a particle ancestry tree to avoid copious amounts of occupancy grid copying (in contrast to FastSLAM where we just maintained a fixed size set of particles and forget old particles).

This creates a new problem: the height of the particle ancestry tree will be linear with respect to the amount of time the algorithm has been running. This means there is no upper bound on the amount of time it takes to examine a cell in an occupancy grid (as we might need to search the entire ancestry tree for changes to the particle). We can solve this problem by carefully defining a method of collapsing certain branches of the tree (Eliazar and Parr [2004]). Notice that any particle that does not produce any surviving children is likely a poor pose estimate and can simply be removed; this may cause a

series of ancestor nodes to be removed as well. Additionally, if a particle has only a single child, the child can be merged with the parent particle and treated as a single particle. This "pruning" technique is demonstrated below:



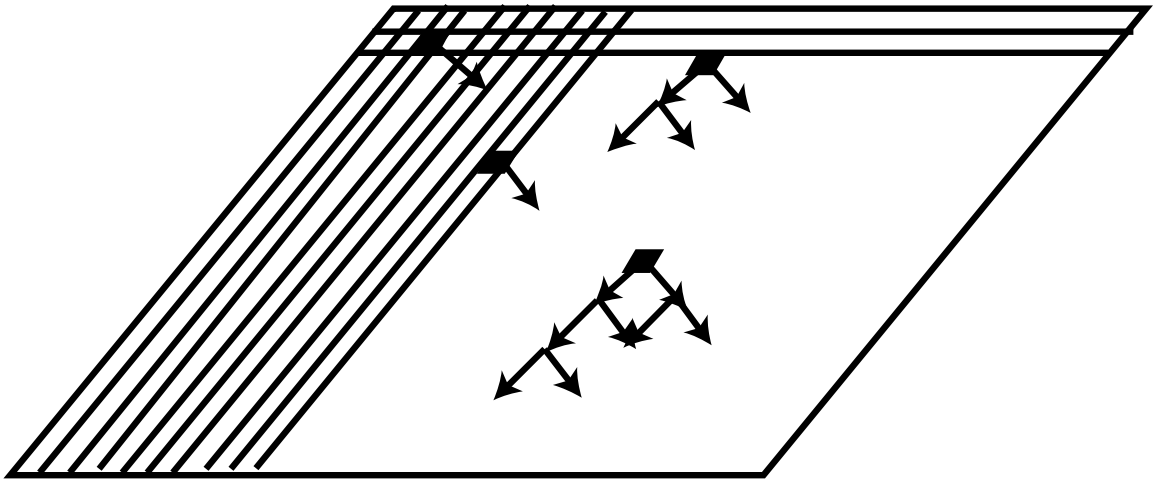
Note that pose3 is crossed out because it has only a single child, but the information from pose3 is not erased but rather it is moved to pose6. If we maintain the particle ancestry tree in this manner we can guarantee that the tree will have a branching factor of at least two, and the depth of the tree will be no greater than the number of particles in each generation (Eliazar and Parr [2004]).

4.4 The Distributed Particle Map

In section 4.3 *A Tree of Particles* we noticed that we could save a lot of time by only remembering how the map of a child particle differs from its parent map. An intuitive way to implement this would be to store a list of changes with each particle specifying which grid cells differ from the parent particle's map. Storing the map information this way causes a bit of a problem when we want make use of the maps. Suppose, for example, the robot wishes to examine the area immediately in front of it to see if there is an obstacle in the way. Even if we only need to consult a single particle, a considerable amount of work is involved. All ancestor particles in the ancestry tree must

be visited, and the list of changes for every particle must be searched for each of the pixels we are concerned about. DP-SLAM solves this problem by developing a distributed particle map.

It seems from the preceding example that it would be nice to have the information pertaining to a particular occupancy grid cell in a predictable place rather than scattered though the ancestry trees of all the particles. This is exactly what the distributed particle map lets us do. The distributed particle map is a single grid with a balanced tree stored in each node (rather than a single occupancy value). The occupancy information from all particles is stored in the trees. The distributed particle map can be visualized as depicted below:



When a particle makes an observation of a grid square, it inserts a unique particle ID into the appropriate tree along with a prediction for occupancy or non-occupancy. If the robot wants to examine a particular pixel the robot now only needs to search through the tree of particles which have actually made an update to that cell (which is likely to be much smaller than an ancestry tree).

At the same time, we can keep the ancestry tree around so we know when we can collapse branches—collapsing branches in the ancestry tree will allow us to also collapse

branches of the distributed particle map's trees. If a child has no siblings and is therefore merged with a parent in the ancestry tree, we can first consult the list of particles the child has changed from its parents map and make the appropriate changes in the distributed particle grid. (The appropriate changes are to alter the parent's observations to match the child's, and then to remove the child's observations from the trees in the distributed particle map.) This bounds the number of objects in each tree to the number of particles in each generation, and in practice the trees remain much smaller (Eliazar and Parr [2004]).

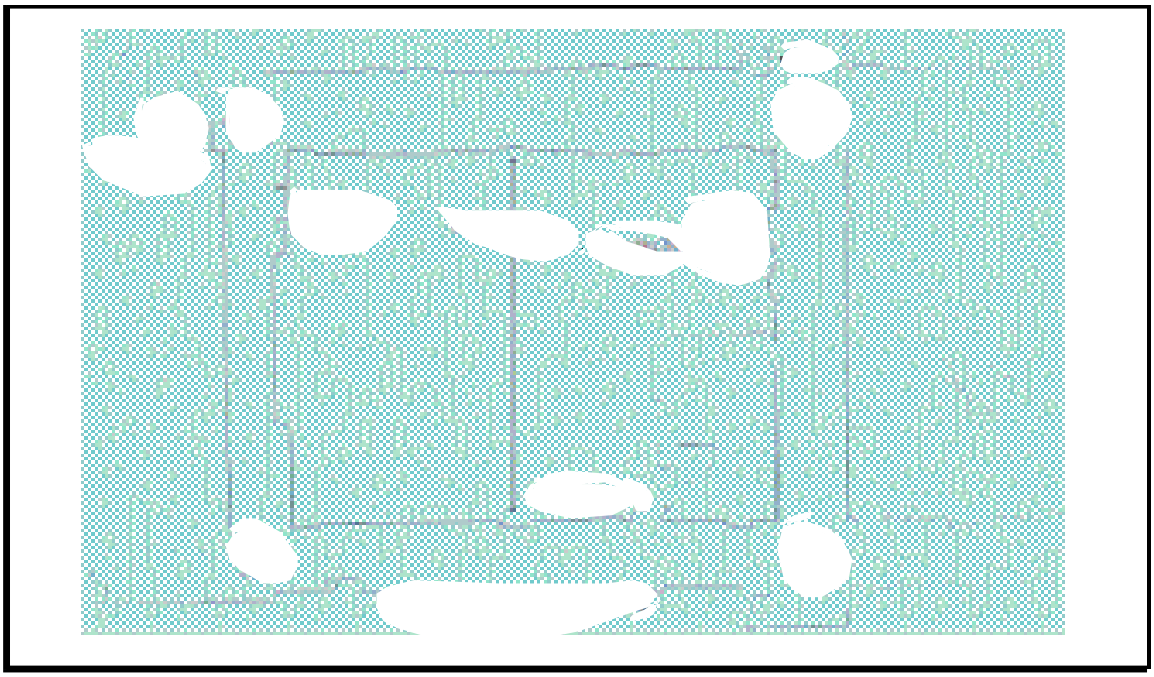
4.5 DP-SLAM in Practice

The computational complexity in terms of strict upper bounds for DP-SLAM may seem larger than we might like. For example, suppose we need to know the prediction made by a particular particle on a particular grid square. In the worst case, we will need to search the entire tree at that grid square for all ancestors of the particular particle. We therefore have a cell access time of O(DlogQ) where D is the height of the ancestry tree and there are Q particles in each generation. Remember we said that the height of the ancestry tree, D, was bounded by the number of particles in each generation, Q. In practice D tends to be approximately O(logQ) (Eliazar and Parr [2004]). The cell access time is therefore approximately $O(\log^2 Q)$.

To generate a new particle, we need to make A cell accesses from the current distributed particle map. (Remember, A is the number of cell observations made by a single pass of the laser.) To produce a new generation of P particles from the last generation of P particles, A * P total cell accesses are required. Since we just decided accessing a single cell takes on the order of $O(\log^2 Q)$, the complexity for generating a

new set of particles is $O(A * P * \log^2 Q)$. We can expect that the cost of inserting information into the distributed particle map and maintaining the particle ancestry tree will have complexity less than or equal to $O(A * P * \log^2 Q)$, so this will ultimately be the time complexity for an iteration of the DP-SLAM algorithm (Eliazar and Parr [2004]).

In practice, the complexity of DP-SLAM is fast enough to be employed in real-time (Eliazar and Parr [2004]). Eliazar and Parr implemented DP-SLAM on a small robot using a resolution of 3 centimeters for the occupancy grid cells, and mapped a 60 meter corridor using 9000 particles. The time it took to collect data for an iteration of the DP-SLAM algorithm was approximately equal to the amount of time it would take a 2.4 GHz computer to perform the computations for the iteration (Eliazar and Parr [2004]). (The robot's processor was slightly slower, so some waiting was involved in the actual experiments at Duke.) DP-SLAM is the only method described in this project which produces maps which are readily interpretable by humans. The following diagram is the complete map corresponding to the highest weighted particle of the 9000 total final generation particles as presented Eliazar and Parr on their original article on DP-SLAM. (It is illuminating to see the product of running the algorithm in a real world environment rather than an example of what a map might look like. I have therefore chosen to include this original diagram produced by the creators of DP-SLAM: Eliazar and Parr. This diagram in no way represents work done by the author of this thesis. Note that this is the only diagram in this thesis copied from another source.)



(Eliazar and Parr [2004]).

As can be seen in the diagram above, DP-SLAM is capable of cleanly mapping a reasonably large area with fairly good accuracy. Remember that, unlike FastSLAM DP-SLAM works without assuming the availability of tens of thousands of trackable features (and in fact without using features at all). Note that the robot makes a complete loop though the corridor and remaps the same areas after traveling approximately 60 meter without sensor contact with these areas. Despite this, we see single distinct lines for walls. This is indicative of the fact that DP-SLAM in general does a very good job of loop closing (Eliazar and Parr [2004]), which we noted was a problem the KF quite often handled poorly. It seems clear that DP-SLAM is a promising solution to the SLAM problem in certain environments—particularly when mapping a two dimensional cross section of the environment is acceptable, and the number of available features is insufficient for other methods.

5 Conclusion

5.1 The SLAM Problem Revisited

We began our discussion in the first chapter by establishing what the SLAM problem is on an intuitive level, and by discussing what a solution to the SLAM problem would accomplish. In short, the SLAM problem is tough to solve or even understand fully, but the closer we get to solving it the more we open the door to a truly autonomous robot. To better understand the SLAM problem, we first broke the SLAM problem down into the component problems (localization and mapping on their own), and we discovered each could be solved without too much trouble. The problem is, in order to localize we need to start with an error free map, and to map we need to be able to localize. This is what motivated us to consider the first solution to the SLAM problem, the Kalman Filter.

5.2 Solutions Revisited

In the Kalman Filter we found a solution that could take advantage of all available data statistically minimize the total squared error in our state prediction. The KF accomplishes this task for us by maintaining a state prediction and a covariance matrix containing a confidence value for every binary relationship in the environment. We noticed the KF specified how each observation could be used to update all feature predictions, and all covariance matrix values. Also, we noticed that the state prediction and covariance matrix had a fixed size independent of the number of iterations of the filter. Consequently, the KF allows us to map for as long as we want without increasing the time to run each iteration. It is safe to say that if there are approximately 200 features we wish to map, a Kalman Filter will be the correct method to use.

While the Kalman Filter is optimal in the sense that it yields a least squares prediction, it cannot always produce this prediction in real time. The problem is maintaining the covariance matrix which is quadratic in size with respect to the number of features. We dealt with this problem in our discussion of FastSLAM. FastSLAM approaches SLAM from a Bayesian perspective and draws upon the insight that observations are conditionally independent of one another given the true robot pose. Errors in robot pose are accounted for by maintaining a set of poses called particles. Our discussion of the KF came in particularly handy here because FastSLAM uses simple Kalman Filters (just like the one we developed at the beginning of chapter 2) to update each feature prediction for each particle. This allows for an approximation of the optimal estimate while at the same time making it possible to take many more features into account. FastSLAM often outperforms KF based approaches despite being non-optimal because so many more features can be utilized in a given amount of time.

We also considered the reverse situation, in which there are not enough features to effectively employ FastSLAM or Kalman Filter based approaches. DP-SLAM uses an entirely different way of representing maps. Laser range finders are employed to create occupancy grids rather than vectors of features. The same insight (conditional independence of observations given robot pose) we discovered in our discussion of FastSLAM is utilized to eliminate the need to correlate observations. Once again, maintaining multiple particles minimizes the effect of errors in the robot pose, and each particle has a corresponding map. The overhead of maintaining multiple occupancy grids is minimized by keeping an ancestry tree and only storing discrepancies between a child an parent. We improved the efficiency in retrieving the occupancy information

corresponding to particular grid cell by distributing the information from all particles over a single particle map. This distributed particle map stores all the information regarding each grid square in a balanced tree for quick lookup.

To summarize: In environments where a couple hundred features are available and it is crucial that the best possible estimate be made for each feature, Kalman Filter approaches outperform the other two. Where tens of thousands of features are available, FastSLAM is usually a better solution. DP-SLAM often excels on level terrain without any features at all. The three solutions we have considered here together represent a good practical solution to the SLAM problem in an environment with any number of features.

5.3 Connections Between Solutions

Earlier in this thesis it was claimed that we would see how these three approaches build off and motivate one another. The basis for this claim should now be clear. It was the high computational complexity of the Kalman Filter which led us to consider a way of approximating the optimal solution without keeping track of the covariance matrix, i.e. FastSLAM.

FastSLAM in turn avoids using the covariance matrix by maintaining multiple pose particles. Not only did the KF motivate FastSLAM, Kalman Filters are actually utilized by FastSLAM to estimate feature locations.

Finally, we realized the motivation for DP-SLAM was the fact that both the KF and FastSLAM require many trackable features which are not always available. DP-SLAM utilizes multiple pose particles (just as we saw in FastSLAM) to improve accuracy.

5.4 Future Work

An extension of the work done in this thesis might be to combine the three algorithms described in this thesis in a way that yields more robust performance. There is no reason a robot would never travel from an environment with many available features to one without features. Additionally a robot might be able to observe both areas with features and areas without from a single location. It seems that if a robot were able to use both a laser range finder to employ DP-SLAM and a camera to employ FastSLAM then it would be better prepared to handle more diverse environments. SLAM methods combining data from camera sensors and laser range finders were considered before FastSLAM or DP-SLAM were developed and these yielded greater accuracy than other contemporary methods (Castellanos, Neira, Tardos, [2001]). It may be that combining the new algorithms we have discussed could yield even better results.

5.5 Concluding Remarks

The time has come to conclude our discussion of the SLAM problem and the promising solutions presented in this thesis. It should now be clear that there is no one right answer to the SLAM problem. There are different ways of collecting data, and different ways of using the data after it is collected. Different environments lend themselves to different approaches, and each of the algorithms presented in this thesis have their place.

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